Mathematics for Computer Science, CM0167,
Example class, Week 10,
Dr David Marshall

1. Pythagoras theorem in $\mathbb{R}^n$: Let $\mathbf{v}$ and $\mathbf{w}$ be two orthogonal vectors in $\mathbb{R}^n$. Show that
$$\|\mathbf{v}+\mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2.$$
NOTE: Be careful! This equality holds only for orthogonal vectors!!!)

2. For which $k \in \mathbb{R}$ are $\mathbf{v}$ and $\mathbf{w}$ orthogonal if,

   a)
$$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 1 \\ 7 \\ k \end{pmatrix}$$

   b)
$$\mathbf{v} = \begin{pmatrix} k \\ k \\ 1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} k \\ 5 \\ 6 \end{pmatrix}?$$

3. Let $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$.

   a) Check wether $\mathbf{c} = \begin{pmatrix} -2 \\ -5 \\ 7 \end{pmatrix}$ is orthogonal to $\mathbf{a}$ or not.

   b) Calculate a non-zero vector that is perpendicular to $\mathbf{a}$ and $\mathbf{b}$.

4. Let P be the parallelogram spanned the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$. Calculate the area of P.

5. Let Q be the parallelepid spanned by the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix}$. Calculate the Volume of Q.

6. Prove the following identities for $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$.

   a) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u}.\mathbf{w})\mathbf{v} - (\mathbf{u}.\mathbf{v})\mathbf{w}$ (Grassmann-expansion).

   b) $(\mathbf{u} \times \mathbf{v}).(\mathbf{w} \times \mathbf{x}) = (\mathbf{u}.\mathbf{w})(\mathbf{v}.\mathbf{x}) - (\mathbf{v}.\mathbf{w})(\mathbf{u}.\mathbf{x})$. (Lagrange identity).