Mathematics for Computer Science, CM0167, Example class, Week 4, Dr David Marshall

- 1. Using the handshaking lemma, show that in any graph, the number of vertices of odd degree is even.
- 2. Consider the following pairs of graphs:



- (a) Are the above graphs *isomorphic*? If yes give a one-to-one correspondence between them. If not show why they can't be isomorphic.
- 3. Find a *trail* and a *path* in the following graphs:



4. *Fleury's algorithm* finds an Eulerian trail in an Eulerian graph:

Algorithm 1 (Fleury's algorithm).

START with an Eulerian graph G.

- Step 1 Choose a starting vertex for your trail.
- Step 2 Starting from the current vertex, traverse any available edge, choosing a bridge only if there is no alternative. Then erase that edge and any isolated vertex. Whenever you erase an edge add it to the trail.
- REPEAT Step 2 until there are no more edges, then STOP.

Use *Fleury's algorithm* to construct an Eulerian trail in the graph below.



5. The Königsberg bridge problem could have been solved if two bridges are removed

Which bridges would you remove? Justify your answer.