Mathematics for Computer Science, CM0167,
Example class, Week 8,

SOLUTIONS
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1. *We are given the following samples of data about the average time a person spends in front of a TV per month.*

$$10,\ 15,\ 22,\ 8,\ 20,\ 29,\ 32,\ 38,\ 4,\ 24,\ 34,\ 28$$

   a) What is the average time for all the people?

   b) Draw a stem-and-leaf-display for the above sample.

a) Average Value is 22

b) Stem-and-leaf-display:

$$\begin{array}{c|ccccc} 0 & 4 & 8 \\ 1 & 0 & 5 \\ 2 & 0 & 2 & 4 & 8 & 9 \\ 3 & 2 & 4 & 8 \end{array}$$

2. *We are given the following samples of data from a measurement of the heights of stairs in office buildings.*

   $9.8cm,\ 8.4cm,\ 8.8cm,\ 7.5cm,\ 10.2cm,\ 9.6cm,\ 9.1cm,\ 10.7cm,\ 7.8cm,\ 8.3cm$

   a) *What is the median value for all the heights of the stairs in the buliding?*

   b) *Draw a stem-and-leaf-display for each of the above samples.*

a) Median

First sort values:

$$7.5\ 7.8\ 8.3\ 8.4\ 8.8\ 9.1\ 9.6\ 9.8\ 10.2\ 10.7$$

There are 10 samples so median value is:

$$(x_5 + x_6)/2 = (8.8 + 9.1)/2 = 8.95.$$

b) Stem-and-leaf-display (Real number so use $\bullet$ for each value in range of integer value):

$$\begin{array}{r|lll}
7 & \bullet & \bullet & \\
8 & \bullet & \bullet & \bullet \\
9 & \bullet & \bullet & \bullet \\
10 & \bullet & \bullet &
\end{array}$$

3. *Consider the ordered sample below*

$$0, 1, 1, 2, 2, 2, 3, 3, 5, 5, 6, 6, 7, 8, 9, 10, 12$$

Calculate the

(a) *mean $\bar{x}$,*

Mean $= 80/16 = 5$

(b) *median x_{med},*

There are 16 samples so mean is:

$$x_{\text{med}} = \frac{1}{2}(x_8 + x_{x_9})$$

which is $(5 + 5)/2 = 5$

(c) *the upper and lower quartile, $x_{0.25}$ and $x_{0.75}$.*

For Lower Quartile:

$n\alpha = 16 \times 0.25 = 4$ which is an integer so

$$x_{0.25} = \frac{x_4 + x_5}{2} = (x_4 + x_5)/2 = (2 + 2)/2 = 2.$$

For Upper Quartile:

$n\alpha = 16 \times 0.75 = 12$ which is an integer so

$$x_{0.75} = \frac{x_{12} + x_{13}}{2} = (x_{12} + x_{13})/2 = (7 + 8)/2 = 7.5.$$

(d) *the inter-quartile range, IQR.*

$$IQR = x_{0.75} - x_{0.25} = 7.5 - 2 = 5.5$$

(e) *the sample variance s^2.*

$$s^2 = \frac{1}{n-1}(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2) = (588 - 16 * 25)/15 = 12.53333333.$$

(f) *the standard skewness* b_3.

$$b_3 := \frac{m_3}{s^3}$$

where s is the sample standard deviation and

m_3 is defined as:

$$m_3 := \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3$$

So

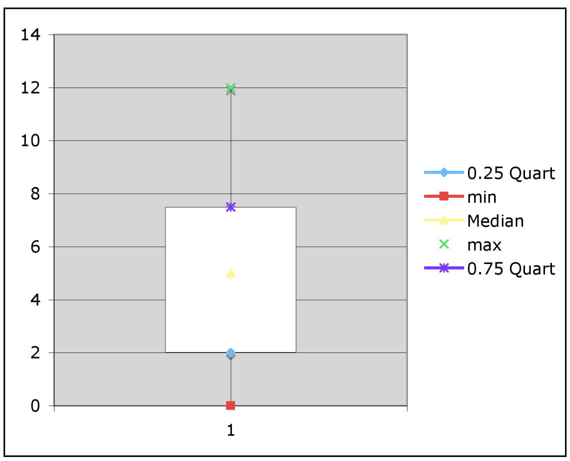
$$m_3 = 15.375$$

So

$$b_3 = (15.375/3.540244813^3) = 0.346509574$$

(g) *Draw a box-plot for the sample.*

The box plot is:



How to draw a box plot in Excel 97/2000/2003 (other versions similar but may have some slight variations):

i. Organise the data as follows in an Excel workbook (The ordering is important):

0.25 Quartile	2
Min	0
Median	5
Max	12
0.75 Quartile	7.5

ii. Highlight the whole table, including figures and series labels, then click on the Chart Wizard.
   - Select a Line Chart.
   - At Step 2 plot by Rows, (the default is Columns), then Finish.

iii. Select each data series in turn and use Format Data Series to remove the connecting lines.

iv. Select any of the data series and Format Data Series:
   - select the Options tab and switch on the checkboxes for High-Low lines and Up-Down bars.

(h) *Calculate the sample variance and the mean of the sample you get after applying the linear transformation*

$$y = 14x - 42$$

From the linear transformation rule: Given the transformed test series $(y_1, \ldots, y_n)$ with $y_i = ax_i + b$ we get $\bar{y} = a\bar{x} + b$ and $s_y^2 = a^2 s_x^2$.
So here $a = 14, b = -42$
so we get

$$\bar{y} = 14 * \bar{x} - 42 = 14 * 5 - 42 = 28$$

and
$s_y^2 = 14^2 \times 12.53333333 = 2456.533333$.

4. *Prove the linear transformation rule: Given the transformed test series* $(y_1, \ldots, y_n)$ *with* $y_i = ax_i + b$ *we get* $\bar{y} = a\bar{x} + b$ *and* $s_y^2 = a^2 s_x^2$.

For $\bar{y}$, substitute for y in definition of the mean $\bar{y}$ (replacing x)

$$\begin{aligned}
\bar{y} &= \frac{1}{n}\sum_i y_i = \frac{1}{n}\sum_i (ax_i + b) \\
&= \frac{1}{n}\left(\sum_i ax_i + \sum_i b\right) \\
&= \frac{1}{n}\left(a\sum_i x_i + nb\right) \\
&= a\frac{1}{n}\sum_i x_i + \frac{1}{n}nb \\
&= a\bar{x} + b
\end{aligned}$$

Q.E.D.

For s_y^2, substitute for y in definition of the mean $\bar{y}$ (replacing x)

$$\begin{aligned}
s_y^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \\
&= \frac{1}{n-1}\sum_{i=1}^{n}(ax_i + b - (a\bar{x} + b))^2 \\
&= \frac{1}{n-1}\sum_{i=1}^{n}(ax_i + b - a\bar{x} - b)^2 \\
&= \frac{1}{n-1}\sum_{i=1}^{n}(ax_i - a\bar{x})^2 \\
&= a^2\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \\
&= a^2 s_x^2
\end{aligned}$$

Q.E.D.

5. *A dice is thrown 4 times. What is the probability that:*

(a) *Four sixes are obtained?*

There $6^4 = 1296$ total possible outcomes.
Four sixes can only occur *once* our all these throws so:

$$P(A = \textit{four sixes thrown}) = \frac{1}{1296}$$

(b) *Four different scores are obtained?*
Four different scores can be obtained $6 \times 5 \times 4 \times 3 = 360$ ways, so

$$P(A = \textit{Four different scores}) = \frac{360}{1296} = \frac{5}{18}$$

6. *A hand of 5 cards is chosen from a standard pack of 52 cards at random. What is the probability that the hand contains:*

   (a) *two spades*

   There are ${}^{52}C_5$ possible hands.

   The number of hands containing 2 spades and 3 non-spades is ${}^{13}C_2 \times^{39} C_3$, so:

   $$P(A = \textit{2 spades and 3 non-spades}) = \frac{{}^{13}C_2 \times^{39} C_3}{{}^{52}C_5} = 0.274$$

   (b) *at least two black cards*

   $$P(A = \textit{at least two black cards} = 1 - P(\textit{0 black cards}) - P(\textit{1 black card})$$

   Now we choose 1 black cards from a possible 26 and 4 red cards from a possible 26:

   $$P(\textit{1 black card}) = \frac{{}^{26}C_1 \times^{26} C_4}{{}^{52}C_5}$$

   Similarly (as all are red):

   $$P(\textit{0 black cards}) = \frac{{}^{26}C_5}{{}^{52}C_5}$$

   So:

   $$P(A = \textit{at least two black cards} = 1 - \frac{{}^{26}C_5 +^{26} C_1 \times^{26} C_4}{{}^{52}C_5} = 0.825$$

7. *(a) Show that* $^mC_n =^m C_{m-n}$.

In general:

$$^nC_k = \frac{n!}{k!(n-k)!}$$

so

$$\begin{aligned} ^mC_{m-n} &= \frac{m!}{k!(m-(m-n))!} \\ &= \frac{m!}{k!(n))!} \end{aligned}$$

Q.E.D.

(b) *Show also that* $^mC_n = \frac{m(m-1)(m-2)...(m-n+1)}{n!}$.

$$\begin{aligned} ^mC_n &= \frac{m!}{n!(m-n)!} \\ &= \frac{m(m-1)(m-2)\ldots(m-n+1).(m-n).(m-n-1)\ldots2.1}{n!.(m-n).(m-n-1)\ldots2.1} \\ &= \frac{m(m-1)(m-2)\ldots(m-n+1)}{n!} \end{aligned}$$

Q.E.D.