**CARDIFF UNIVERSITY**
**EXAMINATION PAPER**

| | |
|---|---|
| **Academic Year:** | 2005/2006 |
| **Examination Period:** | Spring |
| **Examination Paper Number:** | CM0167 |
| **Examination Paper Title:** | Mathematics for Computer Science |
| **Duration:** | 2 hours |

**Do not turn this page over until instructed to do so by the Senior Invigilator.**

**Structure of Examination Paper:**

There are 6 pages.
There are 10 questions in total.
The following appendix is attached to this examination paper on page 5
*CM0167 Exam Formula Sheet*
The mark obtainable for a question or part of a question is shown in brackets alongside the question.

**Students to be provided with:**

The following items of stationery are to be provided:
ONE answer book.

**Instructions to Students:**

Answer all questions.
The use of translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Q1. Find the Huffman code for the character string

’thethreetrees’

and represent it with a binary tree. [7]

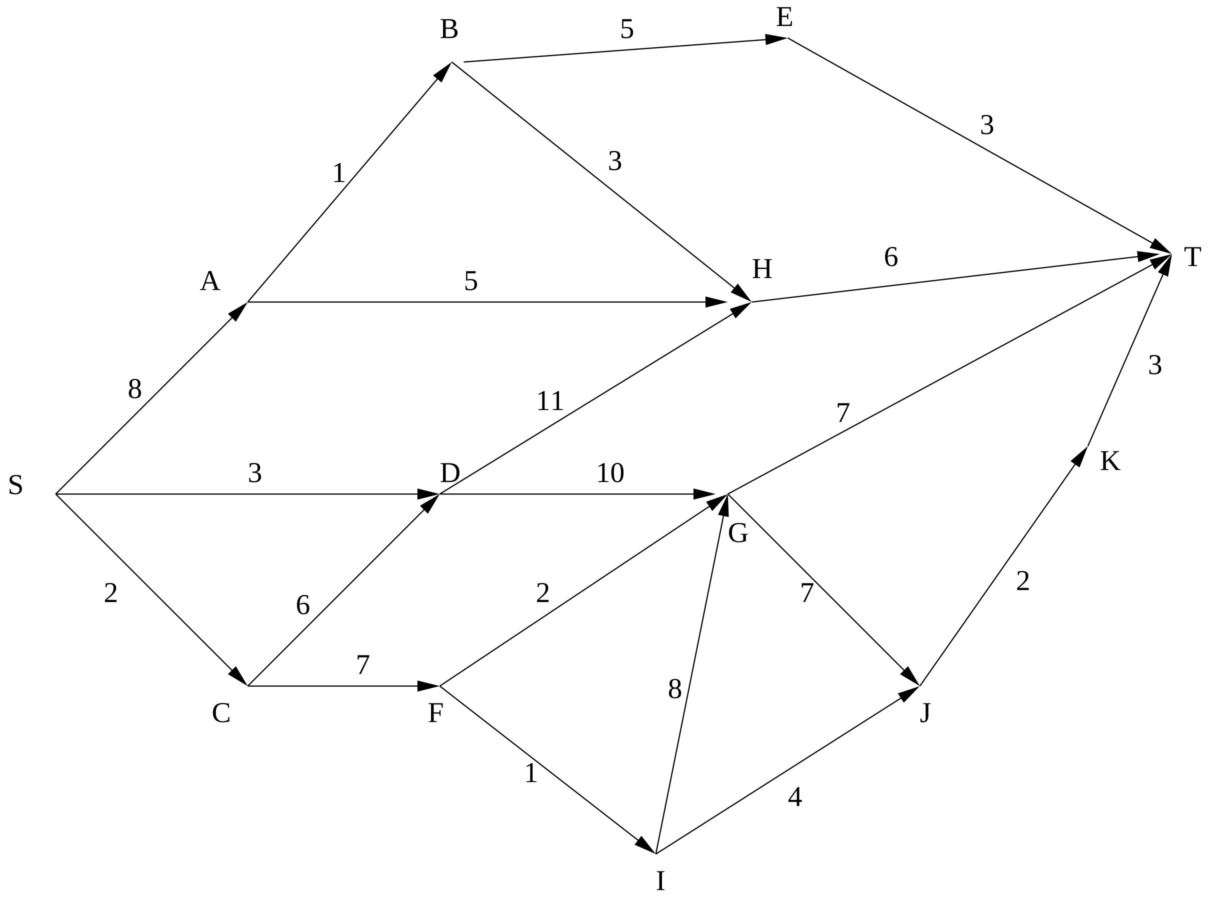
Q2. Consider the following table of distances between the cities $L$, $S$, $C$, $E$, $M$ and $Y$.

| | L | S | C | E | M | Y |
|---|---|---|---|---|---|---|
| L | – | 34 | 28 | 58 | 38 | 29 |
| S | 34 | – | 19 | 36 | 9 | 7 |
| C | 28 | 19 | – | 51 | 35 | 41 |
| E | 58 | 36 | 51 | – | 22 | 47 |
| M | 38 | 9 | 35 | 22 | – | 11 |
| Y | 39 | 7 | 41 | 47 | 11 | – |

a) Find an upper bound for the solution to the travelling salesman problem for the six cities above using the heuristic algorithm. [7]

b) Find a lower bound for the solution to the travelling salesman problem by removing city $L$. [7]

Q3. Find the shortest path from $S$ to $T$ in the digraph below using Dijkstra's algorithm. Show your working with tables. [15]




Q4. A medical disease occurs in $1\%$ of the population. In $8$ out of $10$ cases, where the patient has the disease a new screening procedure gives a positive result. If the patient does not have the disease there is a $5\%$ chance that the procedure still produces a positive result.

a) Draw a tree diagramm for the procedure above with the events $C$ : patient has the disease and $S$ : Screening test gives a positive result. [4]

b) Determine the probability that a randomly selected individual does not have the disease and gives a positive result and that a randomly selected individual gives a positive result on the test. [4]

c) How large is the probability that a person with a positive test result actually got the disease? [4]

Q5. Consider a sample of size 12 about the load of stock funds.

$$0\%,\ 3\%,\ 2\%,\ 2\%,\ 1\%,\ 0\%,\ 1\%,\ 4\%,\ 1\%,\ 0\%,\ 1\%,\ 2\%$$

Calculate the absolute and relative frequency of each load and draw a vertical bar graph for the sample. [5]

Q6. Consider the following sample.

$$0,\ 4,\ 2,\ 5,\ 3,\ 5,\ 7,\ 8,\ 7,\ 9,\ 8,\ 5,\ 5,\ 6,\ 1$$

a) Calculate the arithmetic mean $\bar{x}$ and the sample variance $s^2$. [4]

b) Calculate the inter-quartil-range $IQR$ and the median $x_{med}$ of the sample. [4]

c) Draw a box-plot for the sample. Are there any outliers? [5]

Q7. Consider the following sample of returns on stock funds.

$$4.2\%,\ 1.8\%,\ 9.8\%,\ 6.2\%,\ 0.3\%,\ 2.6\%,\ 7.9\%,\ 5.5\%,$$
$$6.7\%,\ 3.8\%,\ 2.9\%,\ 5.9\%,\ 7.1\%,\ 4.3\%,\ 7.7\%$$

Divide the sample into classes of width $2$ and draw the corresponding histogramm. Make a statement about the modality and the skewness of the histogramm. [6]

Q8. Calculate the volume of the parallelepiped spanned by the three vectors

$$a = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} \quad b = \begin{pmatrix} 3 \\ 0 \\ 8 \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} 1 \\ 5 \\ -8 \end{pmatrix}.$$

[5]

Q9. a) Let $\alpha \in \mathbb{R}$ and $A = \begin{pmatrix} \alpha - 5 & -4 \\ 4\alpha - 2 & 2\alpha \end{pmatrix}$. Find the values for which

$$\det A = 0$$

holds. [4]

b) Calculate the determinant of the matrix

$$B = \begin{pmatrix} -1 & 5 & 3 \\ -2 & 7 & 3 \\ -4 & -1 & -9 \end{pmatrix}$$

[5]

Q10. Calculate the matrix representation of the linear map $f : \mathbb{R}^3 \mapsto \mathbb{R}^3$

$$f(x, y, z) = \begin{pmatrix} \ln(4)z - x \\ 12y + 25z \\ \sin(5)x + 8y - 3z \end{pmatrix}.$$

[6]

**CM0167 Exam Formula Sheet**

**The vector product in $\mathbb{R}^3$:**

The vector product for two three-dimensional vectors $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $w = \begin{pmatrix} w_1 \\ w_2 \\ v_3 \end{pmatrix}$ is given by

$$v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$



END OF EXAMINATION