CARDIFF UNIVERSITY
EXAMINATION PAPER

Academic Year: 2006/2007

Examination Period: Spring

Examination Paper Number: CM0167

Examination Paper Title: Mathematics for Computer Science

Duration: 2 hours

Do not turn this page over until instructed to do so by the Senior Invigilator.

Structure of Examination Paper:

There are 4 pages.
There are 9 questions in total.
There are no appendices.
The mark obtainable for a question or part of a question is shown in brackets alongside the question.

Students to be provided with:

The following items of stationery are to be provided:
ONE answer book.

Instructions to Students:

Answer all questions.
The use calculators **without** programmable memory is permitted.
The use of translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Q1. Apply the *binary tree sort* algorithm to sort the following data

$$8 \quad 2 \quad 9 \quad 12 \quad 6 \quad 4 \quad 5 \quad 1 \quad 9$$

and represent it with a binary tree.

How would you use the tree to sort the data in *ascending* **and** *descending* order?

[6]

Q2. Consider the following table of distances between the cities A, B, C, D, E and F:

	A	B	C	D	E	F
A	–	64	38	28	42	29
B	64	–	27	46	18	9
C	38	27	–	55	25	9
D	28	46	55	–	12	25
E	42	18	25	12	–	31
F	29	9	9	25	31	–

(a) Find an upper bound for the solution to the travelling salesman problem for the six cities above using the heuristic *nearest neighbour* algorithm. [8]

(b) Find a lower bound for the solution to the travelling salesman problem by removing city A. [9]

Q3. Consider the following table of average capacities of communication links in a computer network:

Vertices	*1*	*2*	*3*	*4*	*5*	*6*	*7*
1	–	6	–	2	–	–	–
2	4	–	6	3	2	–	–
3	–	3	–	–	–	1	–
4	2	5	–	–	4	–	–
5	–	3	–	3	–	4	4
6	–	–	3	–	5	–	2
7	–	–	–	–	4	2	–

(a) Represent the above table as *digraph* of the computer network? [4]

(b) Using *Djikstra's algorithm*, Find the shortest path from vertex 1 to all other vertices. Express your solution as a shortest path tree. [9]

(c) Write down the *routing table* for vertex 1. [2]

Q4. Three bags contain red and white balls. Bag 1 contains 8 red and 2 white balls, bag 2 contains 3 red and 4 white balls and bag 3 contains 1 red and 6 white balls.

A person wishes the draw a single ball:

(a) What is the *probability* that a red ball is drawn at random *if all the bags' balls are mixed together*? [3]

(b) What is the *probability* that a red ball is picked *when any one of the bags is first selected at random*? [5]

(c) Given that a red ball as been picked as described in (b) find the *probability that the ball came from bag 2*? [4]

Q5. Consider a sample of size 12 about the load of stock funds.

$$0\%,\ 3\%,\ 1\%,\ 3\%,\ 2\%,\ 1\%,\ 0\%,\ 4\%,\ 0\%,\ 2\%,\ 2\%,\ 1\%$$

Calculate the absolute and relative frequency of each load and draw a vertical bar graph for the sample. [9]

Q6. Consider the following sample.

$$0,\ 9,\ 3,\ 2,\ 4,\ 7,\ 3,\ 4,\ 5,\ 4,\ 3,\ 5,\ 5,\ 5,\ 1$$

(a) Calculate the *arithmetic mean* $\bar{x}$ and the *sample variance* s^2. [5]

(b) Calculate the *inter-quartile range* IQR and the *median* x_{med} of the sample. [4]

(c) Draw a *box-plot* for the sample. Are there any *outliers*? [6]

Q7. Consider the following sample of returns on stock funds.

$$4.5\%,\ 2.8\%,\ 7.8\%,\ 6.5\%,\ 1.3\%,\ 0.6\%,\ 7.3\%,\ 2.5\%,$$
$$4.7\%,\ 3.2\%,\ 4.9\%,\ 6.9\%,\ 7.2\%,\ 4.6\%,\ 8.7\%$$

Divide the sample into *classes of width* 2 and draw the corresponding *histogram*. Make a statement about the *modality* and the *skewness* of the histogram. [8]

Q8. Given the following vectors:

$$\vec{v} = (2, 4), \vec{w} = (1, 6)$$

(a) What are the *norms* of $\vec{v}$ and $\vec{w}$? [2]

(b) What is the *scalar product* $\vec{v}.\vec{w}$? [2]

(c) What is the angle θ between $\vec{v}$ and $\vec{w}$? [3]

(d) What is the *vector cross product* $\vec{v} \times \vec{w}$? [4]

Q9. Calculate the determinant of the matrix

$$B = \begin{pmatrix} -1 & 4 & 2 \\ -2 & 5 & 3 \\ -3 & 0 & -7 \end{pmatrix}$$

[7]