CARDIFF UNIVERSITY
EXAMINATION PAPER

Academic Year: 2007/2008

Examination Period: Spring

Examination Paper Number: CM0167

Examination Paper Title: Mathematics for Computer Science

Duration: 2 hours

Do not turn this page over until instructed to do so by the Senior Invigilator.

Structure of Examination Paper:

There are 5 pages.
There are 9 questions in total.
There are no appendices.
The mark obtainable for a question or part of a question is shown in brackets alongside the question.

Students to be provided with:

The following items of stationery are to be provided:
ONE answer book.

Instructions to Students:

Answer all questions.
The use calculators **without** programmable memory is permitted.
The use of translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Q1. Given the following vertex set, $V = \{A, B, C, D, E\}$, and edge set, $E = \{AB, AE, BC, BD, CE, DE\}$ for a simple graph, $G = (V, E)$:

(a) *Draw* the graph, G. [3]

(b) What is the *order* and *size* of the graph, G [2]

(c) What is the *adjacency matrix* for the graph, G. [3]

Q2. Using the *HuffmanCoding Algorithm* code the following sequence of characters:

`ABBAACCAADDA`

[8]

Q3. : Consider the following table of distances between the cities A, B, C, D and E:

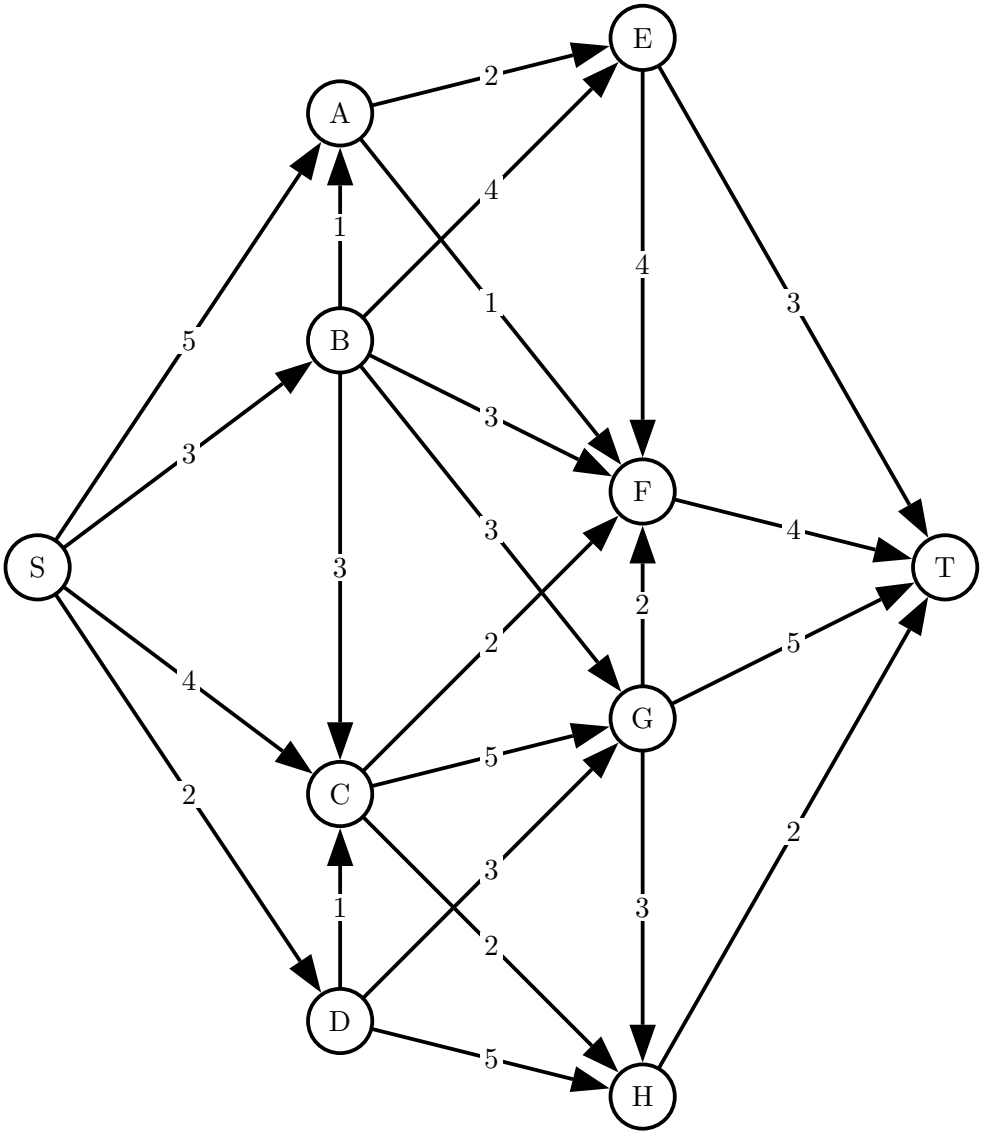
	A	B	C	D	E
A	–	22	20	21	34
B	22	–	47	51	38
C	20	47	–	18	33
D	21	51	18	–	71
E	34	38	33	71	–

(a) Draw a graph to represent the information in the table above. [4]

(b) Find an *upper bound* for the solution to the travelling salesman problem for the six cities above using the heuristic *nearest neighbour* algorithm. [7]

(c) Find a *lower bound* for the solution to the travelling salesman problem by *removing* city A. [7]

Q4. Find the shortest path from S to T in the digraph below using Dijkstra's algorithm. Show your working with tables.



[10]

Q5. A scout group contains 2 adult scoutmasters and 10 boy scouts. They are invited to send four members to a scout convention

(a) Evaluate the *number of ways* that the group may be selected so that it *includes both scoutmasters*? [2]

(b) Evaluate the *number of ways* that the group may be selected so that it *includes only one scoutmaster*? [3]

(c) Evaluate the *number of ways* that the group may be selected so that it *includes neither scoutmaster*? [3]

(d) On a different occasion, the boy scouts decide to play five-a-side football. Each team is chosen by a random selection of all the 10 boy scouts. Given that there are two brothers in the group what is *probability* that the two brothers will be picked in the same team? [5]

Q6. Consider a sample of size 12 about the monthly change in house prices.

$$0\%,\ 1\%,\ 3\%,\ 3\%,\ 2\%,\ 1\%,\ 0\%,\ 1\%,\ 3\%,\ 4\%,\ 2\%,\ 1\%$$

Calculate the *absolute* and *relative frequency* of each monthly change and draw a vertical bar graph for the sample. [9]

Q7. Consider the following sample.

$$0,\ 3,\ 5,\ 2,\ 9,\ 7,\ 3,\ 5,\ 6,\ 4,\ 3,\ 2,\ 4,\ 5,\ 2$$

(a) Calculate the *arithmetic mean* $\bar{x}$ and the *sample variance* s^2. [5]

(b) Calculate the *inter-quartile range* IQR and the *median* x_{med} of the sample. [4]

(c) Draw a *box-plot* for the sample. Are there any *outliers*? [6]

Q8. Given the following vectors:

$$\mathbf{v} = (3, 5), \mathbf{w} = (1, -4)$$

(a) What are the *norms* of $\mathbf{v}$ and $\mathbf{w}$? [2]

(b) What is the *scalar product* $\mathbf{v}.\mathbf{w}$? [2]

(c) What is the angle θ between $\mathbf{v}$ and $\mathbf{w}$? [3]

(d) What is the *vector cross product* $\mathbf{v} \times \mathbf{w}$? [4]

(e) What is the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$? [2]

Q9. Calculate the determinant of the matrix

$$A = \begin{pmatrix} 4 & 2 & 4 \\ -1 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

[7]