CARDIFF UNIVERSITY
EXAMINATION PAPER

Academic Year: 2005/2006

Examination Period: Resit

Examination Paper Number: CM0167

Examination Paper Title: Mathematics for Computer Science

Duration: 2 hours

Do not turn this page over until instructed to do so by the Senior Invigilator.

Structure of Examination Paper:

There are 6 pages.
There are 10 questions in total.
The following appendix is attached to this examination paper on page 5
CM0167 Exam Formula Sheet
The mark obtainable for a question or part of a question is shown in brackets alongside the question.

Students to be provided with:

The following items of stationery are to be provided:
ONE answer book.

Instructions to Students:

Answer all questions.
The use of translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Q1. Find the Huffman code for the character string

'abrakadabrarak'

and represent it with a binary tree. [9]

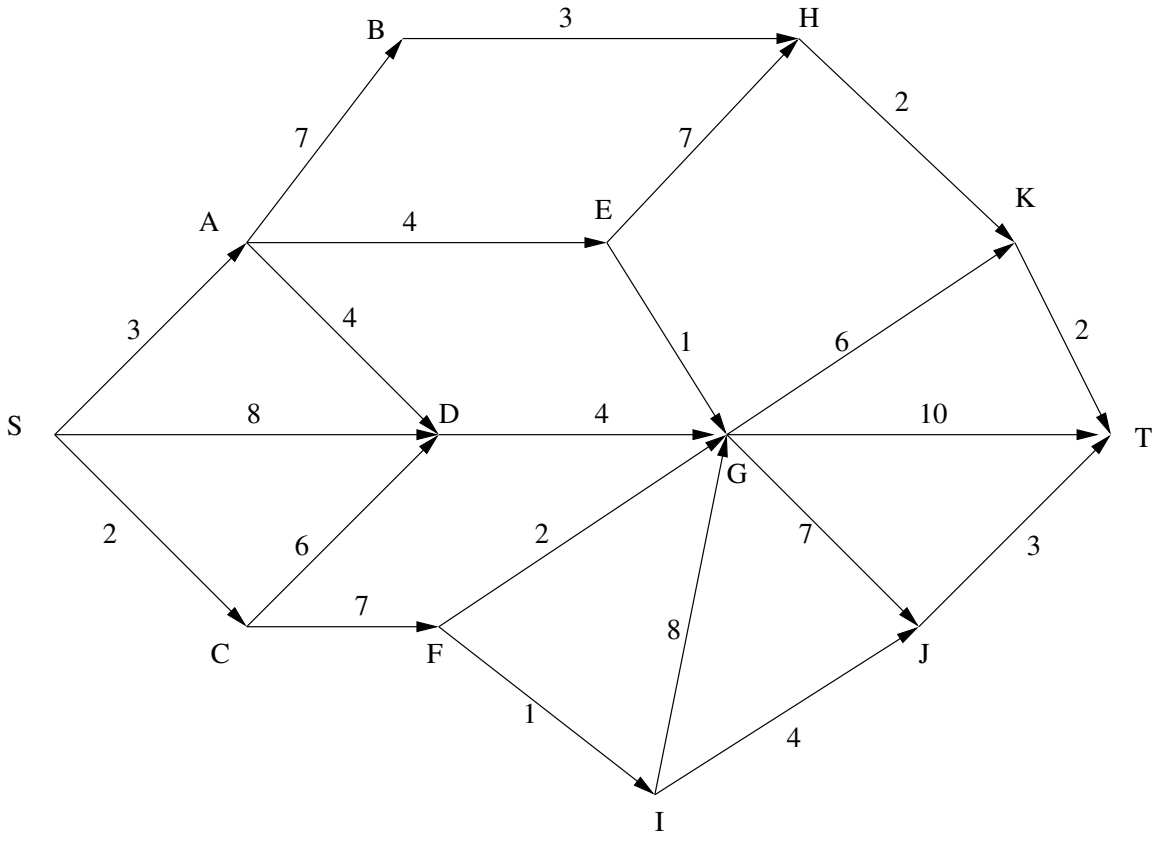
Q2. Consider the following table of distances between the cities L, S, C, E, M and Y.

	L	S	C	E	M	Y
L	–	44	38	68	48	39
S	44	–	29	46	19	17
C	38	29	–	41	25	31
E	68	46	41	–	12	37
M	48	19	25	12	–	11
Y	39	17	31	37	11	–

a) Find an upper bound for the solution to the travelling salesman problem for the six cities above using the heuristic algorithm. [8]

b) Find a minimum connector for the six cities using Prim's algorithm. [6]

Q3. Find the shortest path from S to T in the digraph below using Dijkstra's algorithm. Show your working with tables. [15]



Q4. A medical disease occurs in 1% of the population. In 9 out of 10 cases, where the patient has the disease a new screening procedure gives a positive result. If the patient does not have the disease there is a 10% chance that the procedure still produces a positive result.

a) Draw a tree diagramm for the procedure above with the events C : patient has the disease and S : Screening test gives a positive result. [4]

b) Determine the probability that a randomly selected individual does not have the disease and gives a positive result and that a randomly selected individual gives a positive result on the test. [4]

c) How large is the probability that a person with a positive test result actually got the disease? [4]

Q5. Consider a sample of size 12 on numbers of children in families.

$$0,\ 4,\ 1,\ 1,\ 2,\ 0,\ 1,\ 2,\ 3,\ 3,\ 2,\ 1$$

Calculate the absolute and relative frequency of each load and draw a horizontal bar graph for the sample. [6]

Q6. Consider the following sample.

$$0,\ 4,\ 2,\ 5,\ 3,\ 5,\ 7,\ 8,\ 7,\ 9,\ 8,\ 5,\ 5,\ 6,\ 1$$

a) Calculate the arithmetic mean $\bar{x}$ and the sample variance s^2. [5]

b) Calculate the inter-quartil-range IQR and the median x_{med} of the sample. [4]

c) Draw a box-plot for the sample. Are there any outliers? [6]

Q7. Consider a sample of the heights of stairs in office buildings.

$$9.8cm,\ 8.4cm,\ 8.8cm,\ 7.5cm,\ 6.7cm,\ 9.6cm,\ 9.1cm,$$
$$10.7cm,\ 7.8cm,\ 8.3cm,\ 6.9cm,\ 11.0cm,\ 10.2cm.$$

Draw a stem-and-leaf-display for the sample above. [6]

Q8. Calculate the volume of the parallelepiped spanned by the three vectors

$$a = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad b = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} 0 \\ 5 \\ -5 \end{pmatrix}.$$

[6]

Q9. a) Let $\alpha \in \mathbb{R}$ and $A = \begin{pmatrix} 10 - 2\alpha & 8 \\ 4 - 8\alpha & -4\alpha \end{pmatrix}$. Find the values of α for which

$$\det A = 0$$

holds. [5]

b) Calculate the determinant of the matrix

$$B = \begin{pmatrix} 2 & 5 & -4 \\ -6 & 1 & -7 \\ -4 & 6 & -11 \end{pmatrix}$$

[6]

Q10. Calculate the matrix representation of the linear map $f : \mathbb{R}^3 \mapsto \mathbb{R}^3$

$$f(x, y, z) = \begin{pmatrix} \cos(4)y - z + 12x \\ y - 100x \\ \sin(5)x + 8y - z \end{pmatrix}.$$

[6]

CM0167 Exam Formula Sheet

The vector product in $\mathbb{R}^3$:

The vector product for two three-dimensional vectors $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $w = \begin{pmatrix} w_1 \\ w_2 \\ v_3 \end{pmatrix}$ is given by

$$v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$