## CARDIFF UNIVERSITY EXAMINATION PAPER

Academic Year: ..... 2004/2005
Examination Period: Spring
Examination Paper Number: ..... CM0167
Examination Paper Title: Mathematics for Computer Science
Duration: ..... 2 hours
Do not turn this page over until instructed to do so by the Senior Invigilator.
Structure of Examination Paper:
There are 5 pages.
There are 10 questions in total.
The following appendix is attached to this examination paper on page 5
CM0167 Exam Formula Sheet
The mark obtainable for a question or part of a question is shown in brackets alongsidethe question.

## Students to be provided with:

The following items of stationery are to be provided:
ONE answer book.

## Instructions to Students:

Answer all questions.
The use of translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Q1. Consider the following sample of average over-time hours work per week by civil servants.

$$
0,7,2,6,1,5,5,4,4,3,6,7,2,3,5
$$

a) Calculate the arithmetic mean $\bar{x}$, the sample variance $s^{2}$ and the mode $x_{\text {mod }}$ for the sample above.
b) Calculate the median $x_{\text {med }}$ and the inter-quartil-range $I Q R$ of the sample above.

Q2. The sample below shows the average return of 15 currency stock funds.

$$
\begin{aligned}
& 1.2 \%, 6.8 \%, 7.4 \%, 4.7 \%, 8.1 \%, 0.8 \%, 5.2 \%, 14.8 \% \\
& 7.5 \%, 4.0 \%, 8.4 \%, 4.9 \%, 6.9 \%, 7.8 \%, 8.2 \%
\end{aligned}
$$

a) Construct classes of width $3 \%$ starting with $[0,3 \%)$ and draw a histogramm for the sample. Is the histogram unimodal, bimodal or multimodal?
b) Draw a box-plot for the sample and mark the outliers.

Q3. Let $(\Omega, \mathcal{P}(\Omega), P)$ be a probability space and let $A$ and $B$ be two events.
a) Show that

$$
\begin{equation*}
P(A \mid B)=P(A) \Longleftrightarrow P(B \mid A)=P(B) \tag{5}
\end{equation*}
$$

b) Let $C$ and $D$ be two independent events with $P(C)=\frac{1}{4}$ and $P(D)=\frac{2}{5}$. Calculate

$$
\begin{equation*}
P(C \cup D) \text { and } P\left(C^{c} \mid C \cup D^{c}\right) \tag{6}
\end{equation*}
$$

Hints for the calculation of $P\left(C^{c} \mid C \cup D^{c}\right)$ :

$$
\begin{gathered}
C^{c} \cap\left(C \cup D^{c}\right)=\Omega \backslash(C \cup D) \\
P\left(C \cup D^{c}\right)=P\left(D^{c} \cup(C \cap D)\right)
\end{gathered}
$$

Q4. Tom and Jack play a dice game. Jack throws a dice and Tom wins if the dice shows a 6. They play the game 10 times. Let

$$
X:=\text { Number of games won by Tom }
$$

Calculate $P(X=0), P(X=1), P(X=2)$. What is the probability that Tom wins more than twice? Hint: Use binomial distribution.

Q5. Calculate the volume of the parallelepiped spanned by the three vectors

$$
a=\left(\begin{array}{l}
2  \tag{5}\\
1 \\
4
\end{array}\right) \quad b=\left(\begin{array}{l}
0 \\
1 \\
2
\end{array}\right) \quad \text { and } \quad c=\left(\begin{array}{c}
-1 \\
-1 \\
4
\end{array}\right)
$$

Q6. Solve the linear system $A x=b$ by Gaussian elemination, where

$$
A=\left(\begin{array}{cccc}
2 & -1 & -2 & 0  \tag{8}\\
-4 & 3 & 0 & 2 \\
0 & -3 & 2 & 1 \\
1 & 0 & -4 & 2
\end{array}\right) \quad b=\left(\begin{array}{c}
1 \\
-2 \\
5 \\
1
\end{array}\right)
$$

Q7. a) Let $\alpha \in \mathbb{R}$ and $A=\left(\begin{array}{cc}2 \alpha & 4 \\ 0.5-\alpha & 3 \alpha\end{array}\right)$. Determine the values of $\alpha$ for which $\operatorname{det} A=0$ holds.
b) Calculate the determinant of the matrix

$$
B=\left(\begin{array}{ccc}
-2 & 7 & 6 \\
5 & 1 & -2 \\
3 & 8 & 4
\end{array}\right)
$$

Are the column vectors of $B$ linear independent?
[6]

Q8. a) Calculate the matrix representation of the linear map $f: \mathbb{R}^{3} \mapsto \mathbb{R}^{3}$

$$
f(x, y, z)=\left(\begin{array}{c}
3 x+4 y-5 z \\
-2 y+2 z \\
6 x-\sqrt{2} y-\sqrt{3} z
\end{array}\right)
$$

b) Let

$$
A=\left(\begin{array}{ccc}
2 & -1 & 3 \\
0 & 4 & 5 \\
-2 & 1 & 4
\end{array}\right) \quad \text { and } \quad B=\left(\begin{array}{ccc}
8 & -3 & -5 \\
0 & 1 & 2 \\
4 & -7 & 6
\end{array}\right)
$$

Calculate $A B$.

Q9. Consider the following table of distances between the cities $A, B, C, D, E$ and $F$.

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 56 | 44 | 53 | 41 | 91 |
| B | 56 | - | 82 | 92 | 81 | 76 |
| C | 44 | 82 | - | 34 | 55 | 109 |
| D | 53 | 92 | 34 | - | 107 | 86 |
| E | 41 | 81 | 55 | 107 | - | 121 |
| F | 91 | 76 | 109 | 86 | 121 | - |

a) Find a minimum connector for these cities using Prim's algorithm.
b) Find an upper bound for the solution to the travelling salesman problem for the six cities above using the heuristic-circle-algorithm.

Q10. Find the shortest path from $S$ to $T$ in the digraph below using Dijkstra's algorithm. Show your working with tables.


## CM0167 Exam Formula Sheet

The vector product in $\mathbb{R}^{3}$ :
The vector product for two three-dimensional vectors $v=\left(\begin{array}{l}v_{1} \\ v_{2} \\ v_{3}\end{array}\right)$ and $w=\left(\begin{array}{l}w_{1} \\ w_{2} \\ v_{3}\end{array}\right)$ is given by

$$
v \times w=\left(\begin{array}{l}
v_{2} w_{3}-v_{3} w_{2} \\
v_{3} w_{1}-v_{1} w_{3} \\
v_{1} w_{2}-v_{2} w_{1}
\end{array}\right)
$$

## Binomial coefficients

$$
\binom{m}{n}=\frac{m!}{(m-n)!n!}
$$

## Binomial distribution:

A random variable $X \sim \operatorname{Bin}(n, p)$ - i.e. $X$ is representing the number of successes in $n$ trials, where the probability in each independent trial is $p$ - has the probability mass function

$$
P(X=i)=\binom{n}{i} p^{i}(1-p)^{n-i} \quad i=0,1,2, \ldots, n
$$

