CARDIFF UNIVERSITY EXAMINATION PAPER

Academic Year:	2004/2005	
Examination Period:	Spring	
Examination Paper Number:	CM0167	
Examination Paper Title:	Mathematics for Computer Science	
Duration:	2 hours	

Do not turn this page over until instructed to do so by the Senior Invigilator.

Structure of Examination Paper:

There are 5 pages. There are 10 questions in total. The following appendix is attached to this examination paper on page 5 *CM0167 Exam Formula Sheet* The mark obtainable for a question or part of a question is shown in brackets alongside the question.

Students to be provided with:

The following items of stationery are to be provided: ONE answer book.

Instructions to Students:

Answer all questions.

The use of translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Q1. Consider the following sample of average over-time hours work per week by civil servants.

0, 7, 2, 6, 1, 5, 5, 4, 4, 3, 6, 7, 2, 3, 5

- a) Calculate the arithmetic mean \bar{x} , the sample variance s^2 and the mode x_{mod} for the sample above. [4]
- b) Calculate the median x_{med} and the inter-quartil-range IQR of the sample above. [4]
- Q2. The sample below shows the average return of 15 currency stock funds.

1.2%, 6.8%, 7.4%, 4.7%, 8.1%, 0.8%, 5.2%, 14.8%, 7.5%, 4.0%, 8.4%, 4.9%, 6.9%, 7.8%, 8.2%

- a) Construct classes of width 3% starting with [0, 3%) and draw a histogramm for the sample. Is the histogram unimodal, bimodal or multimodal? [5]
- b) Draw a box-plot for the sample and mark the outliers. [7]
- Q3. Let $(\Omega, \mathcal{P}(\Omega), P)$ be a probability space and let A and B be two events.
 - a) Show that

$$P(A|B) = P(A) \iff P(B|A) = P(B)$$

b) Let C and D be two independent events with $P(C) = \frac{1}{4}$ and $P(D) = \frac{2}{5}$. Calculate

 $P(C \cup D)$ and $P(C^c | C \cup D^c)$

Hints for the calculation of $P(C^c | C \cup D^c)$ *:*

$$C^{c} \cap (C \cup D^{c}) = \Omega \setminus (C \cup D)$$
$$P(C \cup D^{c}) = P(D^{c} \cup (C \cap D))$$

Q4. Tom and Jack play a dice game. Jack throws a dice and Tom wins if the dice shows a 6. They play the game 10 times. Let

X := Number of games won by Tom

Calculate P(X = 0), P(X = 1), P(X = 2). What is the probability that Tom wins more than twice? *Hint: Use binomial distribution.* [5]

[6]

[5]

Q5. Calculate the volume of the parallelepiped spanned by the three vectors

$$a = \begin{pmatrix} 2\\1\\4 \end{pmatrix} \quad b = \begin{pmatrix} 0\\1\\2 \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} -1\\-1\\4 \end{pmatrix}.$$
[5]

Q6. Solve the linear system Ax = b by Gaussian elemination, where

$$A = \begin{pmatrix} 2 & -1 & -2 & 0 \\ -4 & 3 & 0 & 2 \\ 0 & -3 & 2 & 1 \\ 1 & 0 & -4 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ -2 \\ 5 \\ 1 \end{pmatrix}$$
[8]

Q7. a) Let $\alpha \in \mathbb{R}$ and $A = \begin{pmatrix} 2\alpha & 4 \\ 0.5 - \alpha & 3\alpha \end{pmatrix}$. Determine the values of α for which $\det A = 0$ holds. [4]

b) Calculate the determinant of the matrix

$$B = \begin{pmatrix} -2 & 7 & 6\\ 5 & 1 & -2\\ 3 & 8 & 4 \end{pmatrix}$$

Are the column vectors of B linear independent?

a) Calculate the matrix representation of the linear map $f: \mathbb{R}^3 \mapsto \mathbb{R}^3$ Q8.

$$f(x, y, z) = \begin{pmatrix} 3x + 4y - 5z \\ -2y + 2z \\ 6x - \sqrt{2}y - \sqrt{3}z \end{pmatrix}.$$

[5]

[6]

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{pmatrix}$$

Calculate AB.

b) Let

PLEASE TURN OVER

[6]

	Α	В	С	D	E	F
А	-	56	44	53	41	91
В	56	-	82	92	81	76
С	44	82	-	34	55	109
D	53	92	34	-	107	86
Е	41	81	55	107	-	121
F	91	76	109	86	121	-

Q9. Consider the following table of distances between the cities A, B, C, D, E and F.

- a) Find a minimum connector for these cities using Prim's algorithm. [8]
- b) Find an upper bound for the solution to the travelling salesman problem for the six cities above using the heuristic-circle-algorithm. [7]
- Q10. Find the shortest path from S to T in the digraph below using Dijkstra's algorithm. Show your working with tables. [15]



CM0167 Exam Formula Sheet

The vector product in \mathbb{R}^3 :

The vector product for two three-dimensional vectors $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $w = \begin{pmatrix} w_1 \\ w_2 \\ v_3 \end{pmatrix}$ is

given by

$$v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

Binomial coefficients

$$\binom{m}{n} = \frac{m!}{(m-n)!n!}$$

Binomial distribution:

A random variable $X \sim Bin(n, p)$ - i.e. X is representing the number of successes in n trials, where the probability in each independent trial is p - has the probability mass function

$$P(X=i) = \binom{n}{i} p^{i} (1-p)^{n-i} \quad i = 0, 1, 2, \dots, n$$