CM2104: Computational Mathematics Laboratory Worksheet (Week 5)

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Aims and Objectives

After working through this worksheet you should be familiar with:

- Refresh your understanding of some basic **fundamental** mathematics concepts and how these may implemented in MATLAB.
- Understanding polynomials using MATLAB `poly`nomials and MATLAB's symbolic toolbox.

Work through all the questions below slowly, be careful to assimilate the MATLAB and the underlying mathematical ideas. They are essential for understanding of much deeper concepts later in the module.

None of the work here is part of the assessed coursework for this module.

General Maths

This section is meant to be attempted using pen an paper. Giving some practice for exam questions. Solutions will be provided (in one week's time). You can of course use MATLAB (see below) to check your solution.

Polynomials

1. ***Without solving*** the equation, determine the nature of its roots.
   (a) $x^2 - 6x + 4 = 0$
   (b) $3x^2 + 4x + 2 = 0$
2. Given that the quadratic equation $2mx^2 + 2(m+4)x + 9 = 0$ has real roots, show that $m^2 - 10m + 16 \geq 0$.
   (See Further Questions:MATLAB Polynomials Question 6a below)
3. **Curve Sketching**: Find where the following lines and curves cut (i) the y axis (ii) the x axis and then sketch them
   (a) $y = 12 - 2x$
   (b) $y = x^2 + x - 6$

   (See MATLAB Polynomials Question 2 below)

Factorisation

1. Simplify the following:
   (a) $4(3h - 2k) + 5(h + 3k - 1) - 9h$
   (b) $(x + 5)^2$ (by expansion)
2. Factorise the following:
   (a) $6x + 3x^2 + 9xy$
   (b) $x^2 + 8x + 12$
3. **The remainder theorem**: Find the remainder when $6x^3 + 7x^2 - 15x + 4$ is divided by $x - 1$.

MATLAB Polynomials

You may wish to download the MATLAB code demonstrated in the lectures to help you with the following examples. The code is available via learning central (Learning Materials → General Maths → Polynomials) or directly from:
http://www.cs.cf.ac.uk/Dave/CM2104/MATLAB/General_Maths/Polynomials/.

1. Using suitable MATLAB `poly()` and related functions solve to following quadratic equations:

   (a) $2x^2 + 7x + 3 = 0$

   (b) $5x^2 - 4x + 6 = 0$

   (c) ***Now*** Solve the above equations using the MATLAB **symbolic toolbox** functions
   (*See also Further Practice:General Maths:Polynomials Question 2*).

2. Using the MATLAB `poly`**nomial related functions**, define the following functions and plot their graphs:

   (a) $y = 12 - 2x$

   (b) $y = x^2 + x - 6$

   Hint: Look at the convenience function `poly_plot.m`, and examples of how to call it: `poly_plot_eg.m`, discussed in lectures. (Also see General Maths:Polynomials Question 3)

3. Using the **MATLAB symbolic toolbox** define the following functions and plot their graphs:

   (a) $y = 12 - 2x$

   (b) $y = x^2 + x - 6$

   Hint: Look at the example MATLAB function, `line_plot_symbolic_eg.m`, discussed in lectures.

4. **Plotting Asymptotes**: Using the MATLAB symbolic toolbox, define the following functions and plot their graphs with respective asymptotes.

(a) $f(x) = \frac{(x+1)}{(x-1)}$ — *i.e* recreated the figure on page 35 of the Polynomial lecture notes

(b) $f(x) = \frac{2x+1}{(x-2)(x+1)(x-3)}$ — *i.e* recreated the figure on page 38 of the Polynomial lecture notes

Hint: Look at the function `poly_plot.m`, discussed in lectures, which shows how to plot x and y-axes and modify this to plot the Asymptotes.

Polynomials and Factorisation

1. Using the MATLAB symbolic toolbox, simplify the following expressions:

   (a) $4(3h - 2k) + 5(h + 3k - 1) - 9h$

   (b) $\dfrac{8x^2z}{10xyz}$

2. Using the MATLAB symbolic toolbox, factorise the following expressions:

   (a) $6x + 3x^2 + 9xy$

   (b) $3a^2b + 4ab^2 + 2abc$

   (c) $x^3 - 3x^2 - 4x + 12$

3. Use the MATLAB symbolic toolbox to solve the following equation

$$y - \frac{3(y+1)}{5} = 1$$

where

$$y = x^2.$$

Further Practice

General Maths: Polynomials

This section is meant to be attempted using pen an paper. Giving some practice for exam questions. Solutions will be provided (in one week's time). You can of course use MATLAB (see above) to check your solution

1. Without solving the equation, determine the nature of its roots.
   (a) $2x^2 - 5x + 3 = 0$
   (b) $x^2 - 6x + 9 = 0$
   (c) $4x^2 + 12x + 9 = 0$

2. Test whether each of the following could be expressed as the product of two simple linear factors and, where possible, determine those factors.
   (a) $2x^2 + 7x + 3 = 0$
   (b) $5x^2 - 4x + 6 = 0$
   (c) $7x^2 - 5x + 4 = 0$
   (d) $8x^2 + 2x - 3 = 0$

3. Given that the quadratic equation $(1-k)x^2+8x+1-k = 0$ has equal roots, find the possible values of the constant k.
   (See Further Practice: MATLAB Polynomials Question 6b below)

4. Given that the quadratic equation $2x^2 + 8x + k + 3 = 0$ has equal roots, find the possible values of the constant k.

5. For the graph of $y = x^2 + 5x + 4$ find the co-ordinates of
   (a) the points where the line cuts the y-axis
   (b) the point where the line cuts the x-axis

   Hence make a sketch of $y = x^2 + 5x + 4$

6. Prove that for any odd or even function, $f(x)$, $g(x)$ = $f(x)^2$ is always even.

7. Make sketch graphs of the following functions, labelling asymptotes, and giving the coordinates of any intersections with the axes:

   (a) $f(x) = 1 + \frac{3}{x}$
   (b) $f(x) = \frac{2}{x} - 2$
   (c) $f(x) = \frac{4}{x^2} - 1$
   (d) $f(x) = \frac{2-x}{x}$
   (e) $f(x) = \frac{2x-3}{x}$

General Maths: Factorisation

This section is meant to be attempted using pen an paper. Giving some practice for exam questions. Solutions will be provided (in one week's time). You can of course use MATLAB (see above) to check your solution

1. Simplify the following:

   (a) $\frac{8x^2z}{10xyz}$

2. Factorise the following:

   (a) $3a^2b - 4ab^2 + 2abc$
   (b) $h^2 + 2h - 48$
   (c) $5u^2v^2 - 15uv^3 + 10uv^2$
   (d) $x^3 + 2x^2 - 9x - 18$
   (e) $x^3 + x^2 - 10x + 8$
   (f) $x^3 - 10x^2 + 31x - 30$

3. Given that $x+2$ is a factor of the polynomial $x^3+kx^2-4x-12$, where k is a constant.

   (a) show that $k = 3$
   (b) find the other factors of the polynomial.

   (See Further Practice: MATLAB Polynomials Question 6c below)

4. **The remainder theorem**: Find the remainder when:

   (a) $2x^3 - 3x^2 + 5x + 3$ is divided by $x + 1$.

   (b) $x^3 - 7x^2 + 6x + 1$ is divided by $x - 3$.

   (c) $8x^3 - 10x^2 + 7x + 3$ is divided by $2x - 1$.

5. Given that $x+2$ is a factor of $f(x) = x^3 - 7x + k$, where k is a constant.

   (a) show that $k = -6$

   (b) solve the equation $f(x) = 0$

   (c) find the remainder when $f(x)$ is divided by $x + 3$

6. The polynomial $x^3 + 2x + ax + b$ has $x - 1$ as a factor. When the polynomial is divided by $x - 2$, there is a remainder of 12. Find the values of a and b, and the other factors of the polynomial.
   (See Further Practice: MATLAB Polynomials Question 6d below)

MATLAB Polynomials

1. Create a polynomial $f(x) = 3x^3 + 4x^2 - 2x + 1$ as a polynomial structure in MATLAB. Convert this to a MATLAB symbolic expression.

   - Create a MATLAB string for the expression.
     (Hint: Look at the function `poly_plot.m` which shows how to do this).

2. Create a polynomial $f(x) = x^4 + 3x^3 + 4x^2 - 2x + 1$ as a symbolic expression in MATLAB. Convert this to a MATLAB polynomial structure.

   - Create a MATLAB string for the expression.

3. Using the MATLAB symbolic toolbox, solve the following equations

$$5(1 - 2y) = 2(4 - 2y)$$

   where

$$y = 2x + 1$$

4. Using the MATLAB symbolic toolbox, solve the following equations

$$2y^2 - 5y + 2 = 0$$

where

$$y = 3x - 1$$

5. Using the MATLAB symbolic toolbox, factorise the following expressions:

   (a) $5u^2v^2 - 15uv^3 + 10uv^2$

   (b) $x^3 + 2x^2 - 9x + 18$

   (c) $x^4 + x^3 - 7x^2 - x + 6$

6. Using the MATLAB symbolic toolbox:

   (a) Given that the quadratic equation $2mx^2 + 2(m+4)x + 9 = 0$ has real roots, show that $m^2 - 10m + 16 \geq 0$.
   (See General Maths:Polynomials Question 2 above) **Hint: See the MATLAB function** `coeffs()` **to extract coefficients of symbolic polynomial**

   (b) Given that the quadratic equation $(1-k)x^2 + 8x + 1 - k = 0$ has equal roots, find the possible values of k.
   (See Further Practice:General Maths:Polynomials Question 3 above) **Hint: See the MATLAB function** `coeffs()` **to extract coefficients of symbolic polynomial**

   (c) Given that $x + 2$ is a factor of the polynomial $x^3 + kx^2 - 4x - 12$, where k is a constant.

      i. show that $k = 3$

      ii. find the other factors of the polynomial.

   (See Further Practice:General Maths:Factorisation Question 3 above)

   (d) The polynomial $x^3 + 2x + ax + b$ has $x - 1$ as a factor. When the polynomial is divided by $x - 2$, there is a remainder of 12. Find the values of a and b, and the other factors of the polynomial.
   (See Further Practice:General Maths:Factorisation Question 6 above)