CM2104: Computational Mathematics Laboratory Worksheet (Week 6)

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Aims and Objectives

After working through this worksheet you should be familiar with:

- Refresh your understanding of some basic **fundamental** concepts of Calculus .
- Understanding Calculus using MATLAB `poly`nomials and MATLAB's symbolic toolbox.

Work through all the questions below slowly, be careful to assimilate the MATLAB and the underlying mathematical ideas.

None of the work here is part of the assessed coursework for this module.

General Maths: Calculus

This section is meant to be attempted using pen an paper, you may use the CM2104 Maths Formulae sheet (http://www.cs.cf.ac.uk/Dave/CM2104/Maths_Formula_Sheet/Formulae_Sheet.pdf) to help solve the questions below.. Giving some practice for exam questions. Solutions will be provided (in one week's time). You can of course use MATLAB (see below) to check your solution.

Differentiation

1. Differentiate each of the following with respect to x:
   (a) $y = 4x + 2x^2$
   (b) $y = x^{11}$
   (c) $y = x^{-3}$
2. If $y = 2x^2 + 5x - 3$ find the gradient of the curve at the points $x = -1$ and $x = -2$
3. Find the gradient of the following curves at the points indicated:
   (a) $y = x^2 + 3x + 1$ at $(1, 5)$
   (b) $y = \sqrt{x}$ at $(4, 2)$
4. Using the formulae sheet differentiate with respect to x:
   (a) $y = \cos(3x)$
   (b) $y = e^{6x}$
5. Find the co-ordinates of any points on the following lines where the gradient is as stated:
   (a) $y = x^2$, gradient 8
   (b) $y = x^3 + x^2 - x + 1$, gradient zero
6. Find the equations of the tangents at the point indicated:
   (a) $y = \sqrt{x}$ = at (4,2)

7. Find the equations of the normal to the following curves, at the point indicated:

   (a) $y = x^2 - 3x + 4$ at (1,2)

Integration

1. Integrate each function with respect to x:

   (a) $(2x + 3)^{\frac{1}{2}}$

   (b) $\frac{1}{x^5}$

   (c) $2x^5$

   (d) $6x^2 - 4x + 2$

2. Evaluate each of the following definite integrals

   (a) $\int_0^2 x^3 dx$

   (b) $\int_0^2 (x^3 - 3x) dx$

   (c) $\int_1^2 \sqrt{x^5} dx$

MATLAB:Calculus

You may wish to download the MATLAB code demonstrated in the lectures to help you with the following examples. The code is available via learning central (Learning Materials → General Maths → Calculus) or directly from:
http://www.cs.cf.ac.uk/Dave/CM2104/MATLAB/General_Maths/Calculus/.

Differentiation

1. Using MATLAB `poly()` structures differentiate the following:

   (a) $f(x) = 5x^4$

2. Using the MATLAB *Symbolic Toolbox* differentiate the following

(a) $f(x) = 9x + 5$

(b) $f(x) = x^3 + x^2 + x$

3. Find the gradient of the following curves at the points indicated.

   (a) $y = x^2 + 3x + 1$ at $(1, 5)$

   In each case plot the curve and its tangent at the given point in a MATLAB figure.

4. Find the stationary points for the following functions:

   (a) $f(f) = 3x^2 - 2x - 1$

   Determine, whether the points are a *maxima* or a *minima* and plot and label them accordingly in a MATLAB figure.

Integration

1. Using MATLAB `poly()` structures integrate the following:

   (a) $f(x) = 5x^4$

2. Using the MATLAB *Symbolic Toolbox* integrate the following:

   (a) $f(x) = \frac{1}{x^5}$

   (b) $f(x) = \sqrt{x \cos(x)}$

3. Using MATLAB `poly()` structures evaluate the following definite integrals:

   (a) $\int_0^2 x^3 dx$

4. Using the MATLAB *Symbolic Toolbox* evaluate the following definite integrals:

   (a) $\int_2^4 (x^2 + 4) dx$

5. Using the MATLAB, work out the area between the two curves $f(x) = -x^2 + 5x + 15$ and $g(x) = x^2 + 2$

Further Practice

General Maths: Calculus

This section is meant to be attempted using pen an paper. Giving some practice for exam questions. Solutions will be provided (in one week's time). You can of course use MATLAB (see above) to check your solution

Differentiation

1. Differentiate each of the following with respect to x:
   (a) $y = 5x$
   (b) $y = 9x + 5$
   (c) $y = 3x^2$
   (d) $y = 12x^4$
   (e) $y = x^2 + 5x$
   (f) $y = 2 - x^2$
   (g) $y = 3x^2 - 2x - 1$
   (h) $y = x^3 + x^2 + x$
2. Find the gradient of the following curves at the points indicated:
   (a) $y = x^2 + 4x$ at $(0, 0)$
   (b) $y = 5x + x^3x$ at $(-1, -6)$
3. Using the formulae sheet differentiate with respect to x:
   (a) $y = \sin x$
   (b) $y = \sin(2x)$
4. Find the co-ordinates of any points on the following lines where the gradient is as stated:
   (a) $y = x^2 + 3x + 5$, gradient 2
   (b) $y = x^3 + 3x^2 - 5x - 10$, gradient 4

5. Find the equations of the tangents at the points indicated:

   (a) $y = x^2$ at (1,1)

   (b) $y = 2x^2 - 3x + 4$ at (2, 6)

6. Find the equations of the normal to the following curves, at the points indicated:

   (a) $y = \frac{6}{x} + \frac{4}{x^2}$ at (2,4)

Integration

1. Integrate each function with respect to x:

   (a) $(4x + 1)^3$

   (b) x^5

   (c) $3x^2$

2. Evaluate each of the following definite integrals

   (a) $\int_2^4 (x^2 + 4)dx$

   (b) $\int_0^3 (x^2 + 2x - 1)dx$

   (c) $\int_{-1}^0 (3x + 2)^6 dx$

MATLAB: Calculus

Differentiation

1. Using MATLAB `poly()` structures differentiate the following:

   (a) $f(x) = 5x^4$

   (b) $f(x) = 2x^2 + 4x + 1$

2. Using the MATLAB *Symbolic Toolbox* differentiate the following

   (a) $f(x) = 9x + 5$

   (b) $f(f) = 3x^2 - 2x - 1$

(c) $f(x) = x^3 + x^2 + x$

3. Find the gradient of the following curves at the points indicated.

   (a) $y = x^2 + 3x + 1$ at $(1, 5)$

   (b) $y = \sqrt(x)$ at $(4, 2)$

   In each case plot the curve and its tangent at the given point in a MATLAB figure.

4. Find the stationary points for the following functions:

   (a) $f(f) = 3x^2 - 2x - 1$

   (b) $f(x) = x^3 + x^2 + x$

   In each case determine, whether the points are a *maxima* or a *minima* and plot and label them accordingly in a MATLAB figure.

Integration

1. Using MATLAB `poly()` structures integrate the following:

   (a) $f(x) = 6x^2 + 4x + 2$

2. Using the MATLAB *Symbolic Toolbox* integrate the following:

   (a) $f(f) = (4x + 1)^3$

3. Using MATLAB `poly()` structures evaluate the following definite integrals:

   (a) $\int_0^3 x^2 + 2x - 1dx$

4. Using the MATLAB *Symbolic Toolbox* evaluate the following definite integrals:

   (a) $\int_1^2 \sqrt{x^5}dx$

5. Using the MATLAB, work out the area of $f(x) = 3 * x^5 + x^3 - 3$ between $x = -5$ and $x = 5$