CM2208: Scientific Computing
Laboratory Worksheet (Week 2)

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Aims and Objectives

After working through this worksheet you should be familiar with:

- Familiarise yourself with the mathematical foundations and applications of Complex Numbers.
- How to create, manipulate and visualise Complex Numbers in MATLAB;

Work through all the questions below slowly, be careful to assimilate the MATLAB and the ideas behind. They are essential for understanding of much deeper concepts later in the module.

None of the work here is part of the assessed coursework for this module. However, it provides useful basis on which the coursework project can build.

Complex Numbers

MATLAB

Please see the MATLAB Live Script version for a more convenient way to tackle these problems:
http://users.cs.cf.ac.uk/Dave.Marshall/CM2208/Labs/labwk2.mlx

1. Use MATLAB to evaluate:
   (a) $\sqrt{(-25)}$
   (b) $3i + i\sqrt{(2)}$
   (c) $\left(\frac{2}{i}\right)^3$
2. Create the following complex numbers and use MATLAB to simplify (if appropriate) them and find their real and imaginary parts:
   (a) $1 + 2i$
   (b) $(4 - 2i) - (3 + 5i)$
   (c) $(4 - 2i) \times (3 + 5i)$
   (d) $\frac{2+3i}{1-i}$
3. Solve to following equations using MATLAB and use MATLAB to determine if each root is *real* or *complex*.:
   (a) $x^2 - x + 3 = 0$
   (b) $\frac{3x^2}{2} + 15 = 0$
4. Given $z_1 = i - 1$ and $z_2 = 3 - 4i$ using MATLAB
   (a) Draw z_1, z_2, $z1+z_2$, z_1-z_2, $z_1 \times z_2$ and $\frac{z1}{z_2}$ on an Argand diagram.
   (b) Plot the Polar Coordinate versions also
5. Using MATLAB, find $(-1 + i\sqrt{3})^5$.
6. Use MATLAB to create the *phasor* $3e^{135^\circ i}$

(a) Express this phasor in trigonometric form.

(b) Plot the phasor in Polar Coordinate form.

(c) Add the two phasors: $3e^{135°i}$ and $1e^{-25°i}$

Mathematics Examples

Mathematical Fundamentals

Full solutions to maths examples will be provided on Learning Central in Week 3.

1. Simplify:

   (a) $\sqrt{-25}$

   (b) $\sqrt{-125}$

   (c) $(-81)^{\frac{1}{2}}$

2. Find the real and imaginary parts of:

   (a) $1 - 2i$

   (b) $6i$

   (c) $3 + 4i$

   (d) $-\sqrt{3}i$

3. Evaluate:

   (a) $3i - i\sqrt{(2)}$

   (b) $5i + 11i$

   (c) $5i - 7i$

   (d) $3i \times 4i$

   (e) $6i \div 3i$

   (f) $3 + 4i - 5 + 6i$

4. Simplify:

   (a) i^7

(b) i^{-3}

(c) $\left(\frac{2}{i}\right)^3$

(d) $(2-i)^2$

5. Solve the following equations:

   (a) $x^2 + x + 1 = 0$

   (b) $x^2 + x + 3 = 0$

   (c) $\frac{3}{2}x^2 + 15 = 0$

   (d) $2x^2 - 2x + 5 = 0$

6. Express each of the following in the form :

   (a) $\frac{3+i}{4-3i}$

   (b) $\frac{4i}{4+i}$

   (c) $\frac{2+3i}{1-i}$

7. If $z = 4 - 3i$ evaluate:

   (a) z^2

   (b) $\overline{z}$

   (c) $\frac{1}{z}$

8. Solve the following equations for x and y :

   (a) $x + iy = (3+2i)(3-2i)$

   (b) $\frac{x+iy}{2+i} = 5 - i$

The Complex Plane

1. Given $z_1 = -1 + i$ and $z_2 = 3 - 4i$

   (a) draw z_1, z_2 and $z_1 - z_2$ on an Argand diagram.

   (b) find $3z_1 + 2z_2$.

   (c) find $z_1 * z_2$.

   (d) find $z_1 \div z_2$.

Polar Coordintates

1. Find the modulus and argument of each of the following:

   (a) $1 + i$

   (b) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

   (c) $-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

   (d) $-2i$

2. Find the *cartesian co-ordinates* of the points P to S having polar co-ordinates as follows:

   (a) $P(6, -30°)$

   (b) $Q(5, 90°)$

   (c) $R(2, -90°)$

   (d) $S(8, 60°)$

Phasors

1. Simplify: $\mathbf{3}\cos(\omega\mathbf{t} + \mathbf{15}°) + \mathbf{2}\cos(\omega\mathbf{t} + \mathbf{65}°)$

2. Multiply $\mathbf{3}\cos(\omega\mathbf{t} + \mathbf{15}°)$ by the *complex constant*, $2e^{i65°}$.

   Give *one* mathematical interpretation of this operation?