

LIMIT SPECTRAL CURVES FOR THE ORR-SOMMERFELD OPERATOR AND QUASICLASSICAL EIGENVALUE DISTRIBUTION ALONG THESE CURVES

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We study the well-known in hydrodynamics Orr-Sommerfeld problem on the finite interval $[-1, 1]$. We are interested in the question: how do the λ -eigenvalues of this problem behave as the Reynolds number $R \rightarrow \infty$? To answer this question we consider an auxiliary model problem

$$\begin{aligned}i\varepsilon^2 y'' + q(x)y &= \lambda y, \\ y(-1) = y(1) &= 0,\end{aligned}$$

where $\varepsilon = (\alpha R)^{-1/2}$. The limit behaviour of the eigenvalues of this problem as $\varepsilon \rightarrow 0$ depends on analytic properties of $q(z)$, namely, on the topology the Stokes lines, which are defined in the complex plane by the equation

$$\operatorname{Re} S(z) = 0, \quad S(z) = \int_{\xi_\lambda}^1 \sqrt{i(q(\xi) - \lambda)} d\xi,$$

where ξ_λ is the root of $q(\xi) - \lambda = 0$.

We investigate the model problem in two cases: 1) an analytic monotone profile $q(z)$, 2) the so-called Couette–Poiseuille profile $q(z) = az^2 + bz + c$, $a, b, c \in \mathbb{R}$. We prove that the spectrum of the model problem is concentrated along some critical curves in the complex plane, defined by the Stokes lines, which for a monotone profile form a “spectral tie”. We find the formulas for the eigenvalue distribution along the limit curves which are uniform with respect to ε . Then we prove that the limit spectral curves for the original Orr-Sommerfeld problem coincide with those for the model problem, as well as the main terms in the formulas for the eigenvalue distribution along the curves.