DOMAIN GEOMETRY AND EIGENVALUES

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In my lecture I will first present a solution to a question of McKenna and Walter, concerning the positive deformation of hinged plates in a spring bed under positive load. If the spring constant b is small, the corresponding differential operator is positivity preserving, but if it gets larger than a critical constant $b_c(\Omega)$, it is no longer positivity preserving. G.Sweers and I were able to prove that the canonical conjecture $b_c(\Omega) \leq b_c(\Omega^*)$ is false. Here Ω^* is a disc of same area as $\Omega \subset I\!R^2$.

In the second part of my lecture I will address the pseudo-Laplace eigenvalue problem, which comes from minimizing $\sum_{j=1}^{p} \int_\Omega |\partial v/\partial x_j|^p \, dx$ on $K := \{v \in W_0^{1,p}(\Omega \mid ||v||_{L^p(\Omega)} = 1\ \}$. The Euler-Lagrange equation reads

$$\sum_{j=1}^{n} \frac{\partial}{\partial x_j} \left(\left| \frac{\partial v}{\partial x_j} \right|^{p-2} \frac{\partial v}{\partial x_j} \right) + \lambda |u|^{p-2} u = 0,$$

and is more degenerate than $\Delta_p u + \lambda |u|^{p-2} u = 0$. If Ω is a ball, the minimizer is not radially symmetric, but one can still say something about symmetry of the level sets. If Ω is convex, the level sets are convex. These results were obtained jointly with M.Belloni.

If there is time left, I will present results on the symmetry of eigenfunctions in situations where some standard tricks seem to fail.