# Rayleigh-Faber-Krahn inequalities and nonlinear boundary value problems


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The classical Rayleigh-Faber-Krahn inequality states that the among all domains of given volume the first eigenvalue of the membrane is smallest for the ball. This is expressed in terms of Sobolev constants as follows:

$$S_2(D) = \inf_{W_0^{1,2}(D)} \frac{\int_D |\nabla v|^2 dx}{\int_D v^2 dx} \geq S_2(B_1)\mathrm{vol}(D)^{-\frac{N}{2}},$$

where $B_1$ is the unit ball in $\Re^N$. This estimate is obtained by means of Schwarz symmetrization. In this talk we study more general Sobolev constants of the type

$$S_{p,q}(D,a,b) = \inf_{W_0^{1,p}(D)} \frac{\int_D a(x)|\nabla v|^p dx}{\left(\int_D b(x)|v|^q dx\right)^{\frac{p}{q}}},$$

where $a(x)$ and $b(x)$ are positive continuous functions. Without further assumptions on the weights no general Rayleigh-Faber-Krahn inequalities, that is estimates from below depending essentially on $\int_D b(x)dx$, are to be expected. We describe classes of functions $a(x)$ and $b(x)$ for which such inequalities hold. They are then used to construct upper bounds for the solutions of nonlinear boundary value problems involving the p-Laplacian.