ANTI-MAXIMUM PRINCIPLES

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For second order elliptic boundary value problems such as $-\Delta u = \lambda u + f$ in Ω and $u = 0$ on $\partial\Omega$ it is well known that for any reasonable bounded domain in $\mathbb{R}^n$ a first eigenvalue λ_1 exists and moreover, for any $\lambda < \lambda_1$ a positive source term f implies that the solution u is positive. Clément and Peletier showed that for λ in a right neighbourhood of λ_1 opposite behaviour occurs, an phenomenon which they named the *anti-maximum principle*: certain $f > 0$ imply $u < 0$. Such behaviour is roughly explained by the pole of the resolvent at λ_1 and the positive sign of the corresponding eigenfunction φ_1 :

$$u = \frac{1}{\lambda_1 - \lambda} \langle \varphi_1, f \rangle \varphi_1 + R_\lambda f.$$

with $R_\lambda f$ the remaining regular part. A more precise statement reads as: if $0 < f \in L^p(\Omega)$ with $p > n$, then there is $\delta_f > 0$ such that for $\lambda \in (\lambda_1, \lambda_1 + \delta_f)$ the solution satisfies $u < 0$. For higher order elliptic boundary value problems with appropriate boundary conditions sometimes a stronger *uniform anti-maximum principle* holds, that is, for $Lu = \lambda u + f$ in Ω with $Bu = 0$ on $\partial\Omega$ there exists $\delta > 0$ such that for all $\lambda \in (\lambda_1, \lambda_1 + \delta)$ and $f > 0$ one obtains $u < 0$.

The anti-maximum principle for higher order elliptic boundary value problems is joint work with Ph. Clément. Sharp conditions for the uniform anti-maximum principle will appear in a joint work with H.-Ch. Grunau.