

# A PONTRYAGIN SPACE APPROACH TO SINGULAR PERTURBATIONS OF DIFFERENTIAL OPERATORS

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In the study of singularly perturbed differential expressions operators of the form  $A + \langle \cdot, \chi \rangle \chi$  arise, where  $A$  is a self-adjoint operator in some Hilbert space  $\mathcal{H}$  and  $\chi$  is some element of a space with negative norm ( $\chi \in \mathcal{H}_{-k-1} \setminus \mathcal{H}_{-k}$ ) associated with  $A$ . Recently, by Yu. Shondin and J.F. van Diejen/A.Tip a method was developed which associates with the problem a family of operators in some Pontryagin space with negative index  $k$  which are extensions of a symmetric operator in the given Hilbert space. In the lecture an analytic approach to these extensions using the  $Q$ -function or Titchmarsh-Weyl function will be explained. The method applies also e.g. to the Bessel operator  $-\frac{d^2}{dx^2} + \frac{\nu^2 - \frac{1}{4}}{x^2}$  in the self-adjoint case  $\nu > 1$ , if the  $Q$ -function from the non-self-adjoint case  $0 < \nu < 1$  is considered for such values  $\nu > 1$ . Joint work with Y. Shondin and A. Dijkma.