

PRECISE BEHAVIOUR OF THE APPROXIMATION NUMBERS OF THE SOBOLEV EMBEDDING IN THE ONE-DIMENSION CASE

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We consider the Sobolev embedding $I : W^{1,p}(a, b) \rightarrow L^p(a, b)$, $-\infty < a < b < \infty$ and $1 < p < \infty$. We show that for all n in \mathbf{N} the n -th approximation number $a_n(I)$ of I is given by

$$a_n(I) = \frac{c_p}{n}$$

where c_p is a constant dependent only on p . We also find the best n dimensional linear approximation of I and we will discuss the shape of unit ball in $W^{1,p}$.