MONOTONICITY OF EIGENVALUES

LEON GREENBERG
lng@math.umd.edu
Department of Mathematics
University of Maryland
College Park
Maryland MD 20742 , USA

For $0 < y \leq 1$, consider the family of Sturm-Liouville problems:

$$-(p(x)u')' + q(x)u = \lambda r(x)u, \;\; 0 < x < y, \qquad (1a)$$
$$\alpha_0 u(0) + \beta_0 p(0)u'(0) = 0, \qquad (1b)$$
$$\alpha_1 u(y) + \beta_1 p(y)u'(y) = 0, \qquad (1c)$$

where $p \in C^1[0,1],\ q,\ r \in C[0,1]$, and $p,\ r > 0$. The Sturm-Liouville problem (1) has a sequence of eigenvalues: $\lambda_0(y) < \lambda_1(y) < \lambda_2(y) < \cdots$. A well known Sturmian property states that if the second boundary condition (1c) is a Dirichlet condition (i.e. if $\beta_1 = 0$) then the eigenvalues $\lambda_n(y)$ are strictly decreasing functions of y. We ask the question: Does any vestige of this property remain true if $\beta_1 \neq 0$? The answer is yes.

Theorem 1. *There exists an integer $n_0 \geq 0$ such that for $n \geq n_0$, the eigenvalues $\lambda_n(y)$ are strictly decreasing functions of y.*

Theorem 2. *For given functions $p,\ r$ and a given integer $n_0 \geq 0$, there exists a continuous function q such that the eigenvalues $\lambda_n(y),\ 0 \leq n \leq n_0$, are not decreasing. In fact, for any given $y_0 \in (0,1)$, q can be found so that $\lambda_n(y)$ is increasing in a neighborhood of $y_0,\ 0 \leq n \leq n_0$.*