OPTIMAL PRECONDITIONING FOR THE FINITE ELEMENT SOLUTION OF ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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Preconditioning plays an important role in the iterative solution of algebraic problems arising from the numerical discretization of differential operators. It is well known that for many common differential operators the application of standard finite element or finite difference schemes leads to linear systems whose condition number behaves like $O(h^2)$ as $h \to 0$, where h is a typical edge length in the finite element/difference grid. As the grid becomes very fine therefore the importance of preconditioning grows significantly.

A popular class of preconditioning is based upon a geometric decomposition of the grid using additive or multiplicative Schwarz methods. This is because such an approach is naturally amenable to implementation on a parallel computing platform due to the inherent decomposition of the problem. This talk will describe work with three different collaborators, R. Bank, S. Nadeem and S. Nepomnyaschikh, on the development and application of a new variant of a two level additive Schwarz preconditioner that has many of the properties of more complex multilevel preconditioners of the same family. In particular it can be shown that this preconditioner yields a condition number that is independent of the mesh spacing h. Examples of the application of this approach will be provided for problems arising from the discretization of a number of three-dimensional PDEs using linear tetrahedral finite elements.