INVERSE SPECTRAL PROBLEM FOR THE STURM-LIOUVILLE EQUATION, RECOVERY OF THE POTENTIAL FROM THE SPECTRAL DATA

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A method is introduced for the reconstruction of the potential of a Sturm-Liouville operator L generated by $l(y) = -y''(x) + q(x)y(x)$ and separated boundary conditions in $L^1[0, b],\ 0 < b < \infty$ from a spectrum and a set of normalizing constants. These constants can be calculated from the second spectrum with the same boundary conditions at zero.

This method is based on the approach of I. W. Knowles for the recovery of coefficients in a PDE from boundary data and modified for use in this inverse spectral context. We define a functional which is zero at the true potential and calculate its Gateaux derivative in the form $G'(q)[h] = \int_0^1 h(x)H(x)dx$. Then, by the Riesz representation theorem the L^2 gradient is $H(x)$ and we use the gradient descent algorithm to recover the potential in the iterative process.

Since this problem is ill-posed a regularization method is developed to select the best recovered potential. We also discuss results obtained using different types of gradients, behaviour of the method in the presense of noise in the initial data and the implementation details.