HOW ONE CAN COMPUTE WRONG EIGENVALUES AND BELIEVE THEY ARE CORRECT, AND HOW TO REMEDY THEM

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We describe two situations where the convergence of computed eigenvalues is not a guarantee of their exactness.

The first example is the approximation of the eigenfrequencies of an electromagnetic cavity resonator with perfectly conducting walls. The eigenmodes are the eigenvectors of the curl.curl operator, and they are divergence free. Therefore they are also eigenvectors of the regularized operator curl.curl - grad.div, which is nothing but the vector Laplace operator. This corresponds to a coercive variational formulation. Thus, computing eigenpairs by a Galerkin method seems natural. In a quite standard way, one uses finite element approximation. We consider the case when the resonator is polyhedral (or, for simplicity, polygonal). In any situation, the finite element method yields convergence for the lowest eigenvalues. The true eigenvalues are known for a square or a cube. This validates the method. Unfortunately, when the resonator is non- convex, the computed eigenvalues converge to wrong limits, in general. We will explain how one can check the computed eigenvalues, why some of them are wrong, and how to modify the method to remedy the problem.

The second example arises from the linearization of the Ginzburg- Landau operator: the operator under consideration coincides with the Schrödinger operator with magnetic potential. The boundary conditions are natural (Neumann). Like in the semi-classical analysis, there is a small parameter h in front of the principal part of the operator. This small parameter corresponds to a large magnetic field. The eigenmodes of this linearized operator are related to supra-conducting modes. The problem admits a hermitian coercive variational formulation, quite classical, simply scalar. The game is: compute the first eigenvalues in a square for different values of h, and decrease h. If the center of the square coincides with the gauge center, the first obvious observation is that the first eigenvalue tends to h, and that the corresponding eigenmode is a Landau mode, supported around the center of the square. But, in fact, an asymptotic expansion with respect to h tells us that

the first eigenvalue should tend to 0.51 h instead h. We will comment on this and show accurate computations using the p-version of finite elements.