THE INTERIOR TRANSMISSION EIGENVALUE PROBLEM

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In the study of inverse scattering problems one is often led to eigenvalue problems in an interior (i.e. bounded) domain D. For example, in the cace of a scattering problem modeled by the Helmholtz equation in the exterior of D with respect to Dirichlet (or Neumann) boundary condition the eigenvalues of $-\Delta$ in D with respect to homogeneous boundary conditions on ∂D play an important role. These classical eigenvalue problems are well understood with respect to existence, asymptotic behaviour, etc. In the case of an acoustic scattering problem where the scattering medium is penetrable the corresponding time harmonic wave equation takes the form

$$\Delta u \; + \; k^2 n^2 u \; = \; 0 \quad \text{in } \mathbb{R}^3 \, .$$

In this case the values of k for which there exist non-trivial pairs (u, v) of the following homogeneous interior problem play the role of the eigenvalues:

$$\Delta u \; + \; k^2 n^2 u \; = \; 0 \text{ in } D \, , \quad \Delta v \; + \; k^2 \, v \; = \; 0 \text{ in } D \, ,$$

$$u = v \text{ on } \partial D \, , \quad \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} \text{ on } \partial D \, .$$

It is not very much known about this "interior transmission eigenvalue problem". We will give conditions under which the spectrum is discrete and report on a recent existence result by Päivärinta and Sylvester.