# EPSRC Spectral Theory Network Conference III

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## LIST OF PARTICIPANTS

**Dr A Balinsky,** *Electronic mail*: BalinskyA@cardiff.ac.uk

School of Mathematics, Cardiff University, PO Box 926, Cardiff CF24 4YH

**Professor K Ball,** *Electronic mail*: kmb@math.ucl.ac.uk

Department of Mathematics, University College London, Gower Street, London WC1E 6BT

**Professor Dr Catherine Bandle,** *Electronic mail*: bandle@math.unibas.ch

Mathematisches Institut, Universität Basel, Rheinsprung 21, CH-4051 Basel, Switzerland

**Professor M van den Berg,** *Electronic mail*: M.vandenBerg@bristol.ac.uk

School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK

**Dr B M Brown,** *Electronic mail*: Malcolm.Brown@cs.cardiff.ac.uk

Department of Computer Science, Cardiff University, Queen's Buildings, The Parade, PO Box 916, Cardiff CF24 3XF

**V V Burenkov,** *Electronic mail*: V.V.Burenkov@cs.cardiff.ac.uk

Department of Computer Science, Cardiff University, Queen's Buildings, The Parade, PO Box 916, Cardiff CF24 3XF

**Professor B Chanane,** *Electronic mail*: chanane@kfupm.edu.sa

Department of Mathematical Sciences, King Fahd University of Petroleum and Minerals, PO Box 1235, Dhahran 31261, Saudi Arabia

**Professor E B Davies FRS,** *Electronic mail*: e.brian.davies@kcl.ac.uk

Department of Mathematics, King's College, University of London, Strand, London WC2R 2LS

**Professor M S P Eastham,** *Electronic mail*: mandh@chesilhay.fsnet.co.uk

Department of Computer Science, Cardiff University, PO Box 916, Cardiff CF24 3XF, UK

**Dr D Elton,** *Electronic mail*: daniel@ma.hw.ac.uk

Department of Mathematics, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS

**Professor W D Evans,** *Electronic mail*: EvansWD@cardiff.ac.uk

School of Mathematics, Cardiff University, 23 Senghennydd Road, P O Box 926, Cardiff CF24 4YH, UK

**Dr D J Gilbert,** *Electronic mail*: dgilbert@maths.kst.dit.ie

School of Mathematical Sciences, Dublin Institute of Technology, Kevin Street, Dublin 8, Ireland

**Professor I G Graham,** *Electronic mail*: igg@maths.bath.ac.uk

Department of Mathematical Sciences, University of Bath, Bath, BA2 7AY

**M Graham,**

Department of Mathematics, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS

**Professor N Jacob,** *Electronic mail*: N.Jacob@swansea.ac.uk

Department of Mathematics, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP

**M Jais,** *Electronic mail*: M.Jais@cs.cardiff.ac.uk

Department of Computer Science, Cardiff University, Queen's Buildings, The Parade, PO Box 916, Cardiff CF24 3XF

**Professor F Klopp,** *Electronic mail*: klopp@math.univ-paris13.fr

LAGA, Institut Galilée, Université Paris-Nord, F-93430 Villetaneuse, France

**Dr M Levitin,** *Electronic mail*: M.Levitin@ma.hw.ac.uk

Department of Mathematics, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS

**Dr A Liskevich,** *Electronic mail*: V.Liskevich@bristol.ac.uk

Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW

**Dr M Marletta,** *Electronic mail*: MarlettaM1@cardiff.ac.uk

School of Mathematics, Cardiff University, 23 Senghennydd Road, P O Box 926, Cardiff CF24 4YH, UK

**P McKeag,** *Electronic mail*: mckeag@mth.kcl.ac.uk

Department of Mathematics, King's College, University of London, Strand, London WC2R 2LS

**M Lanza de Cristoforis,** *Electronic mail*: mldc@math.unipd.it

Department of Pure and Applied Mathematics, Università degli Studi di Padova, Via G. Belzoni 7, 35131 Padova, Italy

**Dr L Parnovsky,** *Electronic mail*: leonid@math.ucl.ac.uk

University College London, Department of Mathematics, Gower Street, London WC1E 6BT, UK

**Dr A Pushnitski,** *Electronic mail*: A.B.Pushnitski@lboro.ac.uk

Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK

**Dr R V Romanov,** *Electronic mail*: R.Romanov@cs.cardiff.ac.uk

Department of Computer Science, Cardiff University, Queen's Buildings, The Parade, PO Box 916, Cardiff CF24 3XF

**Professor G Rozenblioum,** *Electronic mail*: grigori@math.chalmers.se

School of Mathematical Sciences, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

**Professor Y Safarov,** *Electronic mail*: ysafarov@mth.kcl.ac.uk

Mathematics Department, King's College, University of London, Strand, London WC2R 2LS, UK

**V Samko,** *Electronic mail*: V.Samko@cs.cardiff.ac.uk

Department of Computer Science, Cardiff University, Queen's Buildings, The Parade, PO Box 916, Cardiff CF24 3XF

**Dr R Schilling,** *Electronic mail*: R.Schilling@sussex.ac.uk

School of Mathematical Sciences, University of Sussex, Falmer, Brighton, BN1 9QH

**Dr K M Schmidt,** *Electronic mail*: SchmidtKM@cardiff.ac.uk

School of Mathematics, Cardiff University, Senghennydd Road, Cardiff CF24 4YH, UK

**Dr E Shargorodsky,** *Electronic mail*: E.Shargorodsky@sussex.ac.uk

School of Mathematical Sciences, University of Sussex, Falmer, Brighton BN1 9QH, UK

**Dr A Sobolev,** *Electronic mail*: A.V.Sobolev@sussex.ac.uk

School of Mathematical Sciences, University of Sussex, Falmer, Brighton BN1 9QH, UK

**I Sorell,** *Electronic mail*: i.sorell@lboro.ac.uk

Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK

**Dr A Tyukov,** *Electronic mail*: A.Tyukov@sussex.ac.uk

School of Mathematical Sciences, University of Sussex, Falmer, Brighton, BN1 9QH

**A Veselov,** *Electronic mail*: A.P.Veselov@lboro.ac.uk

Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire LE11 3TU

**Dr R Williams,** *Electronic mail*: Robin.Williams@astro.cf.ac.uk

Department of Physics and Astronomy, Cardiff University, PO Box 913, Cardiff,CF24 3YB, UK

# ABSTRACTS

## HARDY TYPE INEQUALITIES FOR AHARONOV-BOHM MAGNETIC POTENTIALS IN PUNCTURED PLANE

A BALINSKY
BalinskyA@cardiff.ac.uk
School of Mathematics
Cardiff University
PO Box 926
Cardiff CF24 4YH

It is known that classical Hardy inequality does not hold in dimension two. Laptev and Weidl discovered that introducing a magnetic field can improve this situation. In this talk we will present the Hardy type inequalities for magnetic Dirichlet forms with Aharonov-Bohm vector potentials that have multiple singularities.

## ENTROPY JUMPS IN THE PRESENCE OF A SPECTRAL GAP

K BALL
kmb@math.ucl.ac.uk
Department of Mathematics
University College London
Gower Street, London WC1E 6BT

Let $X$ be a random variable with density $f$ satisfying a spectral gap inequality with constant $c$ for the $f$-Laplacian. Let $E_1$ be the entropy $-\int f \log f$ of $X$, let $E_n$ be the entropy of the mormalised sum of $n$ independent copies of $X$ and let $E_\infty$ be the entropy of the Gaussian limit. Then the entropy gap narrows at the (best possible) rate

$$E_\infty - E_n \leq c/n(E_\infty - E_1)$$

This represents the first quantitative estimate of entropy growth for a large class of random variables.

The proof depends upon a new formula for the entropy of a marginal which can be regarded as an inverse to the Brunn-Minkowski inequality.

This formula replaces the problem of estimating entropies of many-fold convolutions with a variational problem which in turn can be tackled using simple spectral methods.

# SIFTING COMPUTED SPECTRA OF EIGENVALUE PROBLEMS ON EXTERIOR DOMAINS

B M BROWN
Malcolm.Brown@cs.cardiff.ac.uk
Department of Computer Science
Cardiff University
Queen's Buildings, The Parade
PO Box 916
Cardiff CF24 3XF

Eigenvalue problems on exterior domains arise in a variety of physical applications: for example, stability of flow around an obstacle, or resonances in partially obstructed waveguides. The numerical treatment of these problems typically generates a number of spurious eigenvalues in addition to the physically correct ones. We show how the Dirichlet to Neumann map can be used to develop a very simple test which identifies the spurious eigenvalues.

This is joint work with M Marletta (Cardiff).

# SPECTRAL POLLUTION

E B DAVIES
e.brian.davies@kcl.ac.uk
Department of Mathematics
King's College
University of London
Strand, London WC2R 2LS

Joint work with Michael Plum. We discuss the determination of eigenvalues of self-adjoint operators for parts of the spectrum in which variational methods cannot be used. Simple-minded methods often give completely wrong 'results'. Our theorems provide a geometrical interpetation of the theorems of Mertins and Zimmerman, and always give rigorous bounds on eigenvalues in gaps, subject to suitable assumptions.

## EIGENVALUE ASYMPTOTICS FOR THE PERIODICALLY PERTURBED HARMONIC OSCILLATOR WITH APPLICATIONS TO THE SPECTRUM OF THE TWO DIMENSIONAL SCHRÖDINGER OPERATOR WITH CONSTANT MAGNETIC FIELD AND PERIODIC ELECTRIC POTENTIAL

D ELTON
daniel@ma.hw.ac.uk
Department of Mathematics
Heriot-Watt University
Riccarton
Edinburgh EH14 4AS

A method is developed to obtain formulae for the high energy asymptotics of the eigenvalues of the one dimensional harmonic oscillator with an additional periodic electric potential. These formulae are applied to the study of the spectrum of the two dimensional Schrödinger operator with a constant magnetic field and periodic electric potential; provided the magnetic flux is rational it is shown that the spectrum is absolutely continuous for sufficiently large energies whenever the electric potential is non-zero and satisfies some extra technical conditions. This is in marked contrast to the situation when the electric potential is zero where it is well known that the spectrum consists of eigenvalues of infinite multiplicity (the so called 'Landau levels').

## COMPUTATION OF EIGENVALUES OF THE LINEARISED NAVIER-STOKES EQUATIONS

I G GRAHAM
igg@maths.bath.ac.uk
Department of Mathematical Sciences
University of Bath
Bath, BA2 7AY

A SPENCE

E VAINIKKO

When steady solutions of complex physical problems are computed numerically it is often crucial to compute their *stability* in order to, for example, check that the computed solution is "physical", or carry out a sensitivity analysis, or help understand complex nonlinear phenomena near a bifurcation point. An important

example is in the computation of fluid flows governed by the steady-state Navier-Stokes equations. Suppose that a velocity field $\mathbf{w}$ has been computed for some particular parameter values. To assess its stability it is necessary to solve the PDE eigenvalue problem:

$$\left.\begin{aligned} -\epsilon\Delta\mathbf{u} + \mathbf{w}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{w} + \nabla p &= \lambda\mathbf{u} \\ \nabla\cdot\mathbf{u} &= 0\ , \end{aligned}\right\} \tag{1}$$

for some eigenvalue $\lambda \in \mathbb{C}$ and nontrivial eigenfunctions $(\mathbf{u}, p)$, satisfying suitable homogeneous boundary conditions. Here the parameter $\epsilon$ is the viscosity, which is inversely proportional to the Reynolds number. The eigenfunctions $\mathbf{u}$ and $p$ are the velocity, a vector field in 2D (resp. 3D) and the pressure $p$ (a scalar field) both defined on a 2D (resp. 3D) domain.

In this talk we describe a numerical algorithm for computing eigenpairs $(\lambda, (\mathbf{u}, p))$ and apply it to some stability problems of contemporary interest. The algorithm includes (i) a mixed finite element discretisation of (1) leading to a large sparse unsymmetric generalised algebraic eigenvalue problem of the form $A\mathbf{x} = \lambda B\mathbf{x}$; (ii) an iterative algorithm for computing a few "dangerous" eigenpairs near the imaginary axis by repeated solution of "shifted" systems of the form $(A-\sigma B)\mathbf{y} = \mathbf{x}$, where $\sigma$ is near an eigenvalue; (iii) the solution of these nearly singular shifted systems by iterative methods based on domain decomposition.

We describe an outline of the mathematical theory of these methods and give illustrations on stability assessment for some discrete Navier-Stokes problems with several hundred thousand degrees of freedom.

## THE NEED OF A SPECTRAL THEORY FOR NON-SYMMETRIC PSEUDO-DIFFERENTIAL OPERATORS WITHOUT PRINCIPAL SYMBOL : MEIXNER-TYPE PROCESSES, NIG-TYPE PROCESSES AND CAUCHY-TYPE PROCESSES

N JACOB
N.Jacob@swansea.ac.uk
Department of Mathematics
University of Wales Swansea
Singleton Park
Swansea, SA2 8PP

The Normal Inverse Gaussian (NIG) processes, the Cauchy process and the Meixner process are Levy processes with characteristic exponents depending on certain parameters, and asymptotically these characteristic exponents are all equi- valent. It is possible to make these parameters state space dependent and then we obtain symbols of pseudo differential operators generating Feller processes. The aim is to get estimates for the corresponding "heat equations", this will lead to estimates for transition probabilities.

The problem is that the symbols for NIG-type and Meixner-type processes do not have a principal symbol, thus the asymptotic comparability with the Cauchy symbol does not give much information. In the talk we will explain these facts more detailed and make first suggestions to overcome some of the problems.

## COMPLEX TUNNELING AND LYAPUNOV EXPONENTS FOR ADIABATIC QUASI-PERIODIC SCHRÖDINGER EQUATION

F KLOPP
klopp@math.univ-paris13.fr
LAGA, Institut Galilée
Université Paris-Nord
F-93430 Villetaneuse
France

The talk is devoted to recent results obtained in collaboration with A. Fedotov (St Petersburg) on the spectral properties of quasi-periodic Schrvdinger operators on the real line. The potential is the sum of a periodic part and of an adiabatic quasi-periodic 'perturbation'. We study the behavior of the Lyapunov exponent with respect to the spectral parameter. It appears that, near the edges of the spectrum of the 'unperturbed' periodic operator, there are energy regions where the Lyapunov exponent becomes abnormally small and oscillates. These oscillations can lead to small 'spots' of the absolutely continuous spectrum in a region where most of the spectrum is singular. We also find strong oscillations of the Lyapunov exponent near relatively small gaps of the unperturbed periodic operator. Here, these oscillations can even lead to the intertwining of small intervals containing essentially only absolutely continuous spectrum and small intervals where the spectrum is singular. We relate these effects to tunneling in the complex phase space and give effective criteria for their occurrence.

## SPECTRAL POLLUTION AND SECOND-ORDER RELATIVE SPECTRA

M LEVITIN
M.Levitin@ma.hw.ac.uk
Department of Mathematics
Heriot-Watt University
Riccarton
Edinburgh EH14 4AS

We consider the phenomenon of spectral pollution arising in calculation of spectra of self-adjoint operators by projection methods. We suggest a strategy of dealing with spectral pollution by using the so-called second order relative spectra. The effectiveness of the method is illustrated by a detailed analysis of two model examples which also show how the standard methods can fail in spectacular fashion even for simplest problems. This is a joint work with E Shargorodsky.

## ON POSITIVE SOLUTIONS TO SEMILINEAR ELLIPTIC INEQIALITIES IN UNBOUNDED DOMAINS

A LISKEVICH
V.Liskevich@bristol.ac.uk
Department of Mathematics
University of Bristol
University Walk
Bristol BS8 1TW

We study the problem of the existence and nonexistence of positive solutions and supersolutions to a semilinear equation of the type $Lu + u^p = 0$, in an unbounded domain $G \subset \mathbb{R}^N$ ($N \geq 3$), where $L$ is a second order elliptic operator. The following simple result is well-known:

If $1 < p \leq \frac{N}{N-2}$ then there are no nontrivial positive solutions to the inequality $\Delta u + u^p \leq 0$ outside a ball in $\mathbb{R}^N$. The value of the critical exponent $p^* = N/(N-2)$ is sharp, i.e. for $p > N/(N-2)$ positive solutions exits.

We discuss different extensions of this result. Namely, we study general second-order differential operators in divergence form with measurable coefficients in place of the Laplacian. Secondly, we consider different geometries of the domain $G$, in particular, we study cone-like domains. We also discuss the case of non-divergence type operators.

## EIGENVALUE ASYMPTOTICS FOR VERY WEAKLY PERTURBED SCHRÖDINGER AND DIRAC OPERATORS WITH CONSTANT MAGNETIC FIELD

G ROZENBLIOUM
grigori@math.chalmers.se
School of Mathematical Sciences
Chalmers University of Technology
SE-412 96 Göteborg
Sweden

It is know that the Schrödinger operator with non-degenerate constant magnetic field has spectrum consisting of eigenvalues with infinite multiplicity, the so called

‘Landau levels’. If the operator is perturbed by an electric potential, tending to zero at infinity, then a sequence of eigenvalues may arise near each Landau level. Earlier, it was found that if the electric field tends to zero at infinity not too fast, e.g., power-like, then asymptotics of these eigenvalues follow quasi-classical pattern and obeys the Weyl-type asymptotical law. In the present talk we consider the case of a compactly supported electric potential and show that here the asymptotic behaviour of eigenvalues is quite pathological, compared with quasi-classical experience. Similar problem is considered for the Dirac operator with constant magnetic field.

## INTEGRABLE SYSTEMS AND SPECTRAL THEORY

A VESELOV
A.P.Veselov@lboro.ac.uk
Department of Mathematical Sciences
Loughborough University
Loughborough
Leicestershire LE11 3TU

A close relation between the theory of integrable systems and spectral theory became clear after the fundamental discovery of the inverse scattering method for the KdV equation in 1960-th. This relation turned out to be very fruitful for both areas. In particular the ideas and methods of integrable systems were used to produce new interesting classes of the Schrödinger operators whose spectrum (or part of it) can be described explicitly. In the talk a review of some results in this direction will be given.