EPSRC Spectral Theory Network Conference IV

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Timetable of Talks and Events

Time	Monday	Tuesday	Wednesday
9:45am	Inequalities in the plane associated with magnetic fields **W D Evans**	Eigenvalues of polygonal drums **L N Trefethen**	Bounds for the Points of Spectral Concentration of One-Dimensional Schrödinger Operators **D J Gilbert**
10:45am	Coffee	Coffee	Coffee
11:15am	Matrix functions: theory and algorithms **N J Higham**	Multidimensional numerical range **Y Safarov**	Eigenvalue curves of circulant Jacobi matrices **B Khoruzhenko**
12:15pm	Lunch	Lunch	Lunch
1:45pm	Absolutely continuous spectrum of many-dimensional Schrödinger operators **A Laptev**	Floquet theory for stability of solitary waves in the Fermi-Pasta-Ulam atomic chain **G Friesecke**	Close
2:45pm	Integrated density of states for the periodic Schrödinger operator **A V Sobolev**	On the existence of minima in the Skyrme model **B Schroers**	
3:45pm	Tea	Tea	
4:15pm	High energy spectral asymptotics of the perturbed two-dimensional magnetic Schrödinger operator **A Pushnitski**	Approximation of semigroups **E B Davies**	
5:15pm	Close	Close	
7:00pm		Banquet	

ABSTRACTS

APPROXIMATION OF SEMIGROUPS

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I show that one can use pseudospectra to construct approximate spectral expansions of highly NSA operators and hence to produce approximate solutions to the evolution equation. These are often much more accurate than what can be obtained using the true eigenvectors of the operators concerned, if, indeed there are any.

INEQUALITIES IN THE PLANE ASSOCIATED WITH MAGNETIC FIELDS

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Hardy and Sobolev-type inequalities in $L^p(\mathbb{R}^2), 1 < p < \infty$, associated with magnetic fields will be discussed, with particular emphasis on the sharpness, or otherwise, of constants. Similar inequalities in the case $p = 2$ for Aharonov-Bohm-type magnetic fields have already been determined by Laptev/Weidl and Balinsky/Evans/Lewis. The lecture will report on joint work with Christer Bennewitz.

FLOQUET THEORY FOR STABILITY OF SOLITARY WAVES IN THE FERMI-PASTA-ULAM ATOMIC CHAIN

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We discuss spectral-theoretic aspects of the stability theory developed recently (in joint work with Robert Pego) for solitary waves in the Fermi-Pasta-Ulam atomic chain.

Solitary waves in discrete particle chains, unlike their continuous counterparts, are not time-independent modulo a spatial shift, but time-periodic modulo a spatial shift.

Hence the linearized evolution equation is

- non-autonomous.

Additional interesting features include:

- the dynamics is Hamiltonian and non-decaying in energy norm; to see 'dispersive decay' one needs to employ carefully chosen weighted norms
- the linearized operator is highly non-normal, possessing continuous spectrum, discrete spectrum, Jordan chains, and no 'spectral theorem'.

We explain how to

- develop a Floquet theory for such systems
- interpret lattice Floquet multipliers / continuous Floquet spectrum as ordinary eigenvalues / ordinary essential spectrum of an associated operator on the real line,
- determine the essential spectrum explicitly, leading to interesting physical conclusions (eg: only supersonic waves can be stable).

Bounds for the Points of Spectral Concentration of One-Dimensional Schrödinger Operators

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We investigate the phenomenon of spectral concentration for one-dimensional Schrödinger operators with decaying potentials on the half-line. Using the Riccati equation to construct appropriate asymptotic series for a generalised Dirichlet m-function, sufficient conditions are established for the existence of first and second derivatives of the spectral function of the operator. This enables explicit estimates for upper bounds of points of spectral concentration to be calculated for suitable classes of short range and long range potentials. An incidental outcome in the case of long range potentials is that estimates of upper bounds for embedded singular spectrum can also be obtained.

This is joint work with B.J. Harris and S.M. Riehl.

Matrix functions: theory and algorithms

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We describe some recent work on matrix functions, covering both theory and algorithms. We discuss general matrix functions, the special case of p^{th} roots, and computation of $f(A)b$ without explicitly forming $f(A)$.

Eigenvalue curves of circulant Jacobi matrices

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Circulant Jacobi matrices are tridiagonal matrices with additional non-zero entries at the right-top and left-bottom corners. This talk present results which show that such matrices have surprising distribution of eigenvalues in the limit of large matrix dimension. Under some general conditions on the matrix entries, the eigenvalues are distributed along analytic arcs in the complex plane and segments of the real axis, and the non-real eigenvalues are regularly spaced. If the matrix entries are chosen randomly then this latter property of the eigenvalue distribution holds with probability one. (Joint work with Ilya Goldsheid).

Absolutely continuous spectrum of many-dimensional Schrödinger operators

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We shall discuss some new results concerning absolute continuity of the spectrum of many-dimensional Schrödinger operators. In order to prove the main statement of the paper we use a reduction to a one-dimensional Schrödinger operator whose potential is an auxiliary operator depending on the spectral parameter. This reduced problem is treated by means of a version of Buslaev-Faddeev-Zakharov's type trace formula.

High energy spectral asymptotics of the perturbed two-dimensional magnetic Schrödinger operator

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We consider the spectrum of the Schrödinger operator in two dimensions with a constant magnetic field and a smooth compactly supported electric potential. If the electric potential is zero, the spectrum consists of the infinitely degenerate eigenvalues (Landau levels). In the presence of the electric potential, there are clusters of eigenvalues near each Landau level. We study the asymptotics of the distribution of eigenvalues in these clusters for high energies. As a by-product of our analysis, we obtain a trace formula for the eigenvalues. This is joint work with Evgeni Korotyaev.

Multidimensional numerical range

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M-dimensional numerical range of an operator A in the Hilbert space H is defined as the set of all M-dimensional vectors whose entries are the values of the corresponding quadratic form on M orthonormal elements of H. We shall discuss the relation between the spectrum of a self-adjoint operator and its multidimensional numerical range. It turns out that the multidimensional numerical range is a convex set whose extreme points are sequences of eigenvalues of the operator A. Every collection of eigenvalues which can be obtained by the Rayleigh–Ritz formula generates an extreme point of the multidimensional numerical range. However, it may also have other extreme points.

On the existence of minima in the Skyrme model

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The Skyrme model is a non-linear theory of nuclear forces in which nuclei are modelled as topological solitons. Alternatively, one may think of it as a non-linear elasticity theory; nuclei correspond to minima of the energy functional satisfying topological boundary conditions. The question of the existence of minima is therefore of great physical and mathematical interest. Earlier work of Esteban established the existence of a minimum in one topological sector of the theory, and gave a sufficient condition for the existence of minima in all sectors. The talk will be about recent joint work with Manton and Singer in which we investigated Esteban's condition.

Integrated density of states for the periodic Schrödinger operator

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The study of various spectral properties for periodic Schrödinger operators is hindered by the presence of the so-called "unstable" eigenvalues in the spectrum. The precision of asymptotic formulae describing the density of states and other spectral quantities eventually depends on how well one knows the structure of the set of these "anomalous" eigenvalues.

The aim of the talk is to present a new high energy asymptotic formula for the density of states of the periodic Schrödinger operator in dimension two. The derivation of this formula relies on a detailed study of the "unstable" sets with the help of the PDO techniques and perturbation theory.

Eigenvalues of polygonal drums

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Many questions of interest to spectral theorists (and physicists) relate to the behavior of eigenvalues and eigenmodes of polygonal drums. New numerical methods have made it possible to explore these questions more easily than before. This talk will hint at the numerics but focus mainly on applications including 'can you hear the shape of a drum?', localization, eigenvalue avoidance, and the design of drums that play chords.