EPSRC Network 'Analysis on Graphs'
UCL Workshop 17-19 Dec 2014
Programme

All talks will take place at:

Department of Mathematics, Room 706, 7th Floor
University College London
25 Gordon Street
LONDON WC1H 0AY

Wed 17 Dec

10.15am	Coffee
11am	Opening
11.15am	Konstantin Pankrashkin *Laplacian and wave equation on equilateral metric graphs*
12.15pm	lunch
1.45pm	Nadia Sidorova *A conditioning principle for Galton-Watson trees*
2.45pm	Coffee
3.15pm	Olaf Post *Approximation of general vertex couplings on quantum graphs by thin branched manifolds*
4.15pm	Ram Band *Anomalous Nodal Count and Singularities in the Dispersion Relation of Honeycomb Graphs*

Thu 18 Dec

9.30am	Cerian Brewer *Wave dynamics on networks of beams/plates*
10.30am	Coffee
11am	Etienne Le Masson *Quantum ergodicity for averaging operators on the sphere*
12noon	lunch
1.30pm	Roman Schubert *Entropy of Eigenfunctions on Quantum Graphs*
2.30pm	Coffee
3pm	Rupert Small *Particle Diagrams and Many-Body Potentials*
4pm	Joachim Kerner *On the instability of Bose-Einstein condensation on quantum graphs under repulsive perturbations*

<u>Fri 19 Dec</u>

9.30am	Chris Joyner *A discrete random walk approach to spectral statistics in Bernoulli matrix ensembles*
10.30am	Coffee
11am	Jonas Søgaard Juul *Flip counts and nodal counts in discrete graphs*
12noon	lunch
1pm	Uzy Smilansky *The spectra of magnetic Laplacian on complete graphs.*

Konstantin Pankrashkin
Laplacian and wave equation on equilateral metric graphs

Abstract:
In this talk I will discuss several questions related to the spectral theory of Laplace operators on infinite equilateral metric graphs. It will be shown that under some choice of boundary conditions at the nodes such operators become unitarily equivalent to some functions of finite-difference operators on the same graph. The machinery will be applied to the study of the averaging operator introduced by Cartwright and Woess and to the proof of a D'Alembert-type formula for the solutions of the wave equation.

Nadia Sidorova
A conditioning principle for Galton-Watson trees

Abstract:
We discuss the behaviour of a Galton-Watson tree conditioned on its martingale limit being small. We prove that it converges to the smallest possible tree, giving an example of entropic repulsion where the limit has no entropy. We also discuss the first branching time of the conditioned tree (which turns out to be almost deterministic) and the strength of the first branching. This is a joint work with N. Berestycki (Cambridge), N. Gantert (Munich), P. Moerters (Bath).

Olaf Post

Approximation of general vertex couplings on quantum graphs by thin branched manifolds

Abstract:

We demonstrate that any self-adjoint coupling in a quantum graph vertex can be approximated by a family of magnetic Schrödinger operators on a tubular network built over the graph. If such a manifold has a boundary, Neumann conditions are imposed at it. The procedure involves a local change of graph topology in the vicinity of the vertex; the approximation scheme constructed on the graph is subsequently ?lifted? to the manifold. For the corresponding operator a norm-resolvent convergence is proved, with the natural identification map, as the tube diameters tend to zero. (Joint work with Pavel Exner)

Ram Band

Anomalous Nodal Count and Singularities in the Dispersion Relation of Honeycomb Graphs

Abstract:

We study the nodal count of the so-called mandarin graphs and show that it exhibits an anomaly: the nodal surplus is never equal to 0 or the first Betti number of the graph. According to nodal-magnetic theorem, this means that bands of the magnetic spectrum (dispersion relation) of such graphs do not have maxima or minima at the usual symmetry points of the fundamental domain of the reciprocal space of magnetic parameters.

In search of the missing extrema we prove a necessary condition for a smooth critical point to happen inside the reciprocal fundamental domain. Using this condition for the mandarin graph with 3 edges, we identify the extrema as the conical singularities in the dispersion relation of the maximal abelian cover of the 3-mandarin, which is the honeycomb graph.

In particular, our results show that the anomalous nodal count is an indication of the presence of the conical points in the dispersion relation of the maximal universal cover. Also, we discover that the conical points are present in the dispersion relation of graphs with much less symmetry than were required in previous investigations.

This is a joint work with Gregory Berkolaiko and Tracy Weyand.

Cerian Brewer
Wave dynamics on networks of beams/plates

Abstract:
We use networks of beams/plates to model noise and vibration in large structur es such as a car.
We begin by considering wave dynamics on one-dimensional beams. This is effectively a quantum graph. However, we use fourth order beam equations in place of the Helmholtz equation. We use a decomposition to write the full solution in terms of two waves; one approaching and one leaving the boundary. This decomposition will be illustrated and a generalisation to higher dimensions discussed. The ultimate goal is to treat higher dimensional quantum graph problems.
Wave dynamics on networks of two-dimensional plates are then considered, as outlined in Langley and Heron's "Elastic wave transmission through plate/beam junctions". We are now interested in relating the incident and outgoing power via a scattering matrix. In Langley and Heron's paper, it is assumed that evanescent modes do not carry power. Hence only propagating response waves are considered. Assuming that evanescent modes can carry power, we wish to incorporate evanescent modes into the formulation of the scattering matrix. Due to the inclusion of evanescent modes, the problem will be non-unitary. We can, however, guarantee that the secular equation will have real roots.

Etienne Le Masson
Quantum ergodicity for averaging operators on the sphere

Abstract:
The quantum ergodicity theorem (Snirelman, Zelditch, Colin deVerdiere) says that on a compact Riemannian manifold with ergodic geodesic flow, for any orthonormal basis of eigenfunctions of the Laplacian in L^2, the modulus squared of these eigenfunctions converge weakly as measures to the uniform measure in the limit of large eigenvalues, up to a density 1 subsequence. On the sphere the geodesic flow is not ergodic and it is possible to find basis of eigenfunctions that don?t satisfy the conclusion of the theorem. However, Zelditch has proved that it holds almost surely for random eigenbasis.We will present a quantum ergodicity theorem on the sphere for joint eigenfunctions of the Laplacian and an averaging operator over a finite set of rotations. The proof also brings a new argument for quantum ergodicity on regular graphs. Joint work with Shimon Brooks and Elon Lindenstrauss.

Roman Schubert
Entropy of Eigenfunctions on Quantum Graphs

Abstract:
We are interested in the distribution of eigenfunctions on quantum graphs, in particular how they depend on the topology of the graph. As a measure for the distribution we consider the entropy; if an eigenfunction has a large entropy it implies that it cannot be concentrated on a small set of edges. We will focus on two classes of graphs, star graphs and regular graphs. For star graphs we show that the average of the entropies of eigenfunctions is small, indicating eigenfunctions which localise on few bonds. In contrast for regular graphs with large girth we show that the entropy of eigenfunctions is large. The strongest estimates we obtain for expanders where we choose the length of the bonds randomly, then we can show that with large probability the entropy is at least half as large as the maximal possible value. This is analogous to the results by Anantharaman and Nonnenmacher on the entropy of quantum limits on Anosov manifolds, and we in particular borrow one of their tools, the entropic uncertainty principle by Maassen and Uffink.

Rupert Small
Particle Diagrams and Many-Body Potentials

Abstract:
k-Body Potentials, of which spin hypergraphs and embedded random matrix ensembles are special cases, are hamiltonians specified by a sum of interaction terms between k-tuples of nodes on a graph. Focusing on embedded random matrix potentials a diagrammatic technique involving "particle diagrams" will be introduced to calculate moments of the level density for these systems.
The essence of the problem is actually one of summing over all sets which satisfy some very basic overlapping conditions (e.g. the overlap between sets A and B equals C). The relationship between sets which overlap "optimally" can be seen using diagrams, and in this way a tricky summation over several overlapping sets can be translated into a diagram, which can in turn be translated into an asymptotic answer to the question "how many sets satisfy overlapping condition X?"

Joachim Kerner

On the instability of Bose-Einstein condensation on quantum graphs under repulsive perturbations

Abstract:

To prove the existence of Bose-Einstein condensation (BEC) in interacting systems is generally considered a hard problem and only recently results have been obtained. On the other hand, in order to see that condensation is not a purely exotic and theoretical phenomenon, it is equally important to understand the effect of interactions on BEC in systems that exhibit condensation on a non-interacting level.

In this talk, we discuss Bose-Einstein condensation in interacting many-particle systems on general quantum graphs and extend previ- ous results obtained for particles on an interval. We show that even arbitrarily small repulsive two-particle interactions destroy condensa- tion which is present in the non-interacting Bose gas. Most impor- tantly, our results cover singular two-particle interactions such as the well-known Lieb-Liniger model. (This talk is based on joint work with J. Bolte).

Chris Joyner

A discrete random walk approach to spectral statistics in Bernoulli matrix ensembles

Abstract:

We develop a new approach for obtaining the eigenvalue statistics of large random Bernoulli matrices, (matrices in which the elements are chosen from a binary set with equal probability). Specifically we initiate a discrete random walk process in the space of such ensembles and analyse the resulting motion in the reduced space associated to the eigenvalues. In the limit of large matrix size we show how this motion approaches the well known Dyson Brownian motion for Gaussian ensembles and thus we are able to recover the joint probability density function for the eigenvalues via a suitable Fokker-Planck equation.

Jonas Søgaard Juul
Flip counts and nodal counts in discrete graphs

Abstract:
The existence of non-isomorphic graphs which share the same Laplace spectrum (to be referred to as cospectral graphs) leads naturally to the following question: What additional information is required in order to resolve cospectral graphs? It was suggested this might be achieved by counting the number of nodal domains or sign flips in the corresponding eigenfunctions. Here we provide examples to the contrary, by describing three types of cospectral graphs that share the same number of nodal domains or sign flip counts. In addition, we provide a mechanism and give examples of non-cospectral graphs that also have the same nodal counts and sign flip counts.

Uzy Smilansky
The spectra of magnetic Laplacian on complete graphs.

Abstract:
The magnetic Laplacian is obtained from the standard graph Laplacian, by decorating each non vanishing entry $A_{r,s}$ of the adjacency matrice by a unitary number$e^{i\phi_{r,s}}$, with $\phi_{s,r} = -\phi_{r,s}$ and $\phi_{r,r} = 0$. The lecture will focus on the ensemble of Laplacians obtained by choosing the phases independently and uniformly from $[0, 2\pi]$, where the basic (non-magnetic) adjacency is that of complete graphs. Several more and less traditional methods will be presented, and the remaining problems to be addressed in the future will be discussed. (Work done with Hans Weidenmüller and Chris Joyner).