## Dynamic data resolution to improve the tractability of UMTS network planning

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Dynamic data resolution to improve the tractability of UMTS network planning

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Abstract

Transmission site selection and configuration for cellular networks is an NP-hard optimisation problem. Consequently efforts to improve tractability are very valuable and meta-heuristic algorithms are now commonly applied. The speed of configuration evaluation is a binding constraint on performance of meta-heuristic techniques. This is particularly challenging for CDMA-based systems because power allocation is required before coverage can be evaluated. The current most efficient heuristic for achieving this requires at worst $O(n_{cell}^2 \cdot n_{stp})$ time where $n_{cell}$ is the number of cells and $n_{stp}$ is the number of active user locations and $n_{stp}$ is large compared to $n_{cell}$. Consequently in large-scale scenarios, tractability is significantly impeded by $n_{stp}$.

We introduce a new approach to improve tractability for cell planning. This concerns changing the resolution of the problem scenario by creating virtual entities which combine spatial traffic requirements, thus reducing the number of decision variables (i.e., $n_{stp}$ is reduced). We examine in detail the change in quality of solution that this method induces. The results show that only a marginal reduction in quality of network evaluation while efficiency is improved.

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Keywords: Cell planning, network evaluation, power control, radio network design, UMTS, CDMA.

1 Introduction

Cellular networking planning involves selecting and configuring base station sites and transmission equipment so that coverage and capacity requirements are met. This is formally recognized as a computationally complex (NP-hard) problem [10] and accordingly optimization models and techniques for automatic planning have become increasingly important. This is particularly the case for third generation (3G) systems based on WCDMA, such as UMTS.

To manage complexity, meta-heuristic algorithms are now commonly applied involving evaluation of a large number of candidate configurations. Examples of this include genetic approaches [4], tabu search [9] and simulated annealing [6]. The performance of these approaches depends on evaluation speed and this motivates our investigation.

As explained in [8], the use of snapshots is the most common approach to modeling the spatial and temporal traffic demand. For CDMA-based systems such as UMTS, power allocation to transmitters is required before coverage evaluation can take place. Effective evaluation can be performed in $O(n_{cell}^2 n_{stp})$ time [12], where $n_{cell}$ is the number of active cells and $n_{stp}$ are the number of user locations where service is requested. Typically $n_{cell}$ is small relative to $n_{stp}$.

In this paper we introduce a novel mechanism to improve tractability for network planning. The approach effectively changes the resolution of the problem scenario by introducing the concept of the virtual stp. A virtual stp represents a number of ‘local’ stp that are combined, reducing the number of $n_{stp}$ that require evaluation. This is beneficial for practical large-scale network planning because it reduces complexity for the evaluation techniques (as developed in [12]) and heuristics where network evaluation is required (e.g., [4, 9, 6]).

Our focus is to investigate the concept of the virtual stp and to determine the effects on accuracy when applying it. To do so, we:
• formally introduce the virtual stp and its supporting theory;

• specify mathematical optimization models to determine optimal coverage in test scenarios;

• evaluate the difference in accuracy from applying different parameterizations of virtual stp as compared to modeling problems only with stp.

These results allow us to determine the effect of using virtual stp and establishes their benefit when combined with evaluation techniques such as those in [12] and heuristics requiring repeated network evaluation (e.g., [4, 9, 6]).

2 Model

Our model concerns the downlink scenario. We use the standard modeling convention where at a single point in time, a picture of possible requests for downlink services is taken. This snapshot is composed of test points representing physical locations at which service quality is tested. Each test point is either a pilot test point (ptp) or a service test point (stp). At a ptp, adequate pilot signal strength and quality is required. In addition, at an stp a particular downlink UMTS service is required. The set of all ptp and all stp is denoted $S_{ptp}$ and $S_{stp}$ respectively.

Candidate base station sites are locations at which multiple antennae could operate. A single base station configuration specifies a base station site and the configuration of a transmitting antenna at that site (e.g., height, tilt, direction). The set of all permitted base station configurations for a problem scenario is denoted by $S_t$. This represents the search space from which candidate base station configurations are selected. The path-loss between a base station configuration $j$ and a test point location $i$ is denoted $PL_{ij}$. Given a particular base station configuration, the received power $p_{ij}$ at test point $i$ from a base station configuration $j$ is:

$$p_{ij} = P_{ij} - TL_{ij}$$

where $P_{ij}$ is the transmission power of transmitter $j$ for serving test point $i$ and $TL_{ij}$ is the total loss
defined by:

\[ TL_{ij} = L^v_{ij} + L^h_{ij} + PL_{ij} - G^t_j \]  

(2)

where \( G^t_j \) is the antenna gain of base station configuration \( j \), \( L^v_{ij} \) is the loss in the vertical plane due to antenna tilt of the base station configuration \( j \) as received at test point \( i \), \( L^h_{ij} \) is the loss in the horizontal plane due to antenna direction of base station configuration \( j \) as received at test point \( i \). In this paper path loss \( PL_{ij} \) is estimated using the empirical Hata model [5, 2] as commonly used.

2.1 UMTS load and coverage

The total power resource available at base station configuration \( j \) is denoted \( P^{max}_j \). The proportion of power dedicated for pilot channels remains fixed. The total power dedicated to traffic and pilot channels should not exceed some proportion \( \eta_{max,j} \) of \( P^{max}_j \), and thus \( \eta_{max,j} \) represents the maximum load permitted at \( j \).

**Definition 1.** A ptp \( i \) is covered by base station configuration \( j \) if both:

\[ \frac{p^{pilot}_{ij}}{N + \sum_{k \in S^*_t} I_{ik}} \geq (E_c/I_o)_{pilot} \]  

(3)

\[ \frac{p^{pilot}_{ij}}{N + \sum_{k \in S^*_t} I_{ik}} \geq (E_c/I_o)_{pilot} \]  

(4)

where \( \pi \) is the threshold on the minimum required power for decoding at the handset, \( N \) is the background noise, \( S^*_t \) is the subset of base station configurations that are commissioned, \( p^{pilot}_{ij} \) is the received pilot power at ptp \( i \) from base station configuration \( j \), \( I_{ij} \) is the total received transmission power (on pilot and traffic channels) from base station configuration \( j \), and \( (E_c/I_o)_{pilot} \) is the required target threshold for pilot \( E_c/I_o \) ratio.

**Definition 2.** An stp \( i \) is covered by base station configuration \( j \) if equations 3 and 4 are satisfied and also:

\[ \frac{W}{R_i} \cdot \frac{p_{ij}}{N + I_{ij}(1-\alpha) + \sum_{k \in S^*_t \setminus j} I_{ik}} \geq (E_k/N_o)_{R_i} \]  

(5)

\[ \frac{W}{R_i} \cdot \frac{p_{ij}}{N + I_{ij}(1-\alpha) + \sum_{k \in S^*_t \setminus j} I_{ik}} \geq (E_k/N_o)_{R_i} \]  

(6)
where: $p_{ij}$ is the received traffic channel power assigned for stp $i$ from base station configuration $j$; $W$ is the CDMA chip rate; $R_i$ is the data rate of the service requested by stp $i$; $\alpha$ is the orthogonality factor and $(E_b/N_o)_{R_i}$ is the threshold determined by the data rate of the service $R_i$.

The general objectives of the UMTS network planning problem are two-fold: (i) maximize the downlink coverage ratio, which is the proportion of stp covered; (ii) minimize commitment to infrastructure provision (e.g., minimize the size of $S^*_t$ as defined in Definition 1). These are conflicting objectives.

3 Problem Formulation

To investigate the use of virtual stp, we formulate two discrete mathematical optimization problems to model coverage evaluation. Two problems are considered: maximizing coverage while selecting a given number of base station configurations from a set $S_t$ of candidates (Section 3.1) and maximizing coverage for a given selection of base station configurations $S^*_t \subseteq S_t$ (Section 3.2). For $i \in S_{stp}$ and $j \in S_t$, we define the following binary variables:

$$x_{ij} = \begin{cases} 
1 & \text{if stp } i \text{ is covered by base station configuration } j \text{ as in Definition 2} \\
0 & \text{otherwise.} 
\end{cases}$$

$$y_j = \begin{cases} 
1 & \text{if base station configuration } j \text{ has been selected from } S_t \text{ for operation} \\
0 & \text{otherwise.} 
\end{cases}$$

The proportion of maximum power $P_{j}^{max}$ allocated at base station configuration $j$ to support the traffic channel for stp $i$ is denoted $P_{ij}$.
3.1 Maximizing coverage while selecting a given number of base station configurations

\[
\text{Maximize } \sum_{i \in S_{\text{stp}}} \sum_{j \in S_i} x_{ij} \left/ |S_{\text{stp}}| \right. \\
\text{subject to the following constraints:}
\]

\[
\sum_{j \in S_i} x_{ij} \leq 1 \forall i \tag{7}
\]

\[
x_{ij} \leq y_j \forall i, j \tag{8}
\]

\[
\sum_{j \in S_i} y_j \leq K_1 \tag{9}
\]

\[
\eta_j = \sum_{i \in S_{\text{stp}}} P_{ij} + y_j T_{\text{pilot}} \leq \eta_{\text{max},j} \forall j \tag{10}
\]

\[
P_{ij} \leq x_{ij} \forall i, j \tag{11}
\]

\[
\sum_{j \in S_i} (T_{\text{pilot}} C_{ij}) x_{ij} \geq \pi \sum_{j \in S_i} x_{ij} \forall i \tag{12}
\]

\[
\sum_{j \in S_i} (T_{\text{pilot}} C_{ij}) x_{ij} + \infty (1 - \sum_{j \in S_i} x_{ij}) \geq (E_b/N_0) R_i (R_i/W) [N + \sum_{k \in S_i} (\eta_k C_{ik})] \forall i \tag{13}
\]

\[
P_{ij} C_{ij} + \infty (1 - x_{ij}) \geq (E_b/N_0) R_i (R_i/W) [N + (\eta_j C_{ij})(1 - \alpha) + \sum_{k \in S_i \setminus j} (\eta_k C_{ik})] \forall i, j \tag{14}
\]

where:

\[
C_{ij} = 10^{(P_{j}^{\text{max}}-T_{L_{ij}}-30)/10} \tag{16}
\]

and \(T_{\text{pilot}}\) is the fixed power assigned at each base station configuration for pilot coverage. Note that \(C_{ij}\) is pre-computable and represents the strength of maximum transmission power as received at \(\text{stp} i\) from transmitter \(j\).

Constraint (7) ensures that an \(\text{stp}\) will be served by at most one base station configuration. Constraint (8) ensures that a base station configuration must be commissioned before any \(\text{stp}\) is served by it. Constraint (9) ensures that no more than \(K_1\) base station configurations will be commissioned. Constraint (10) ensures that the total power allocated at a base station configuration including the pilot power must not exceed \(\eta_{\text{max},j}\). Constraint (11) ensures that \(P_{ij}\), the power assigned to base station con-
configuration \(j\) for serving \(stp\ i\), is non-zero only if \(stp\ i\) is served by \(j\). Constraints (12) and (13) together ensure that \(stp\ i\) will be pilot covered by base station configuration \(j\) only if: (i) received pilot power at \(stp\ i\) from base station configuration \(j\) is more than \(\pi\); and (ii) the signal to interference plus noise ratio as received at \(stp\ i\) from base station configuration \(j\) based on fixed pilot power \(T^{\text{pilot}}\) is more that the required threshold \((E_c/I_o)_{\text{pilot}}\). Constraints (14) and (15) together ensure that \(stp\ i\) will be served by base station configuration \(j\) only if: (i) \(stp\ i\) is pilot covered by \(j\); (ii) received signal strength at \(stp\ i\) from base station configuration \(j\) is more than \(\pi\), and (iii) the signal to interference plus noise ratio as received at \(stp\ i\) from base station configuration \(j\) is more that the required threshold \((E_b/N_o)_{R_i}\). The number of binary variables in the model is \(nm + m\), where \(n\) and \(m\) are the cardinality of \(S_{\text{stp}}\) and \(S_t\) respectively. The number of real variables is \(nm\), and the number of constraints is \(4nm + 3n + m + 1\).

### 3.2 Maximizing coverage for a given selection of base station configurations

Let \(S_t^*\) be a given set of base station configurations that are all operational.

Maximize \[
\frac{\sum_{i \in S_{\text{stp}}} \sum_{j \in S_t^*} x_{ij}}{|S_{\text{stp}}|}
\]

subject to the following constraints:

\[
\sum_{j \in S_t^*} x_{ij} \leq 1 \quad \forall i \tag{17}
\]

\[
\eta_j = \sum_{i \in S_{\text{stp}}} P_{ij} + T^{\text{pilot}} \leq \eta_{\text{max},j} \quad \forall j \tag{18}
\]

\[
P_{ij} \leq x_{ij} \quad \forall i, j \tag{19}
\]

\[
\sum_{j \in S_t^*} (T^{\text{pilot}} C_{ij}) x_{ij} \geq \pi \sum_{j \in S_t^*} x_{ij} \quad \forall i \tag{20}
\]

\[
\sum_{j \in S_t^*} (T^{\text{pilot}} C_{ij}) x_{ij} + \infty (1 - \sum_{j \in S_t^*} x_{ij}) \geq (E_c/I_o)_{\text{pilot}} [N + \sum_{k \in S_t^*} (\eta_k C_{ik})] \quad \forall i \tag{21}
\]

\[
P_{ij} C_{ij} \geq \pi x_{ij} \quad \forall i, j \tag{22}
\]

\[
P_{ij} C_{ij} + \infty (1 - x_{ij}) \geq (E_b/N_o)_{R_i} \left(\frac{R_i}{W}\right) [N + (\eta_j C_{ij})(1 - \alpha) + \sum_{k \in S_t^* \setminus j} (\eta_k C_{ik})] \quad \forall i, j \tag{23}
\]
where:

\[ C_{ij} = 10^{(P_{\text{max}}^j - TL_{ij} - 30)/10} \]  

Constraints (17) - (23) act as those described in Section 3.1. However because the given set of base station configurations in \( S_t^* \) are all operational there is no \( y_j \) variable in any constraint.

4 Changing Data Resolution: The Virtual stp

The basis of our approach is that “similar” stp may be combined to form virtual stp. These collectively capture the same total traffic requirement for a planning scenario and we show that careful constraints in the specification of virtual stp permit good coverage estimations to be made for the original problem. Introducing virtual stp reduces the number of test points in the problem specification, reducing complexity in the underlying model.

Definition 3. A virtual stp \( V \) is a subset of \( S_{stp} \).

With respect to a particular base station configuration \( j \), the specification of the total loss for a virtual stp is dependent on the context in which the total loss is being used. Consequently for a virtual stp \( V \) we define:

\[ TL_{Vj}^{\text{wanted}} = \max\{TL_{kj} : k \in V\} \]  

\[ TL_{Vj}^{\text{unwanted}} = \min\{TL_{kj} : k \in V\} \]  

If \( V \) is a virtual stp then for the calculation of \( p_{Vj} \) and \( p_{Vj}^{\text{pilot}} \), \( TL_{Vj}^{\text{wanted}} \) is used. For the calculation of \( I_{Vj} \), \( TL_{Vj}^{\text{unwanted}} \) is used. A virtual stp \( V \) is pilot covered by base station configuration \( j \) if both:

\[ p_{Vj}^{\text{pilot}} \geq \pi \]  

\[ \frac{p_{Vj}^{\text{pilot}}}{N + \sum_{j \in S_t^* I_{Vj}}} \geq \frac{(E_c/I_o)_{\text{pilot}}}{N} \]  

where \( \pi \) is the threshold on the minimum required power for decoding at the handset, \( N \) is the background noise, \( S_t^* \) is the set of commissioned base station configurations. A virtual stp is \( V \) is covered by base
station configuration \( j \) if Equations 27 and 28 are satisfied and also:

\[
p_{Vj} \geq \frac{\sum_{k \in V} R_k (E_b/N_0) R_k}{V_{\text{min}}}, \pi \quad (29)
\]

\[
\frac{P_{Vj}}{N + I_{Vj}(1-\alpha) + \sum_{k \in S^*_j \setminus j} I_{Vk}} \geq \sum_{i \in V} \frac{R_i (E_b/N_0) R_i}{W} \quad (30)
\]

where:

\[
V_{\text{min}} = \min\{\frac{R_i (E_b/N_0) R_i}{W} : i \in V\} \quad (31)
\]

The model presented in Sections 3.1 and 3.2 remain valid for virtual \( \text{stp} \) as for \( \text{stp} \) provided that constraints 12, 13, 14 and 15 in Section 3.1 and 20, 21, 22 and 23 in Section 3.2 are updated in accordance with Equations 27, 28, 29 and 30 respectively. This is trivial and introduces no further decision variables to the problem formulation.

Lemma 1. If a virtual \( \text{stp} \) \( V \) is pilot covered by a base station configuration \( j \) then all \( \text{stp} \) \( i \in V \) are also pilot covered by \( j \).

Proof: Each \( \text{stp} \) \( i \in V \) must satisfy Equations 3 and 4. Equation 3 is necessarily satisfied because \( T_{L_{Vj}}^{\text{wanted}} \geq T_{L_{ij}} \) and so \( p_{ij}^{\text{pilot}} \geq p_{Vj}^{\text{pilot}} \geq \pi \). Additionally \( \sum_{k \in S^*_i} I_{ik} \leq \sum_{k \in V} I_{Vk} \) since \( \forall j T_{L_{Vj}}^{\text{unwanted}} \leq T_{L_{ij}} \). Hence Equation 4 is satisfied. \( \square \)

Lemma 2. If a virtual \( \text{stp} \) \( V \) is covered by a base station configuration \( j \) then all \( \text{stp} \) \( i \in V \) are also covered by \( j \).

Proof: For each \( i \in V \), let:

\[
\gamma_i = \frac{R_i (E_b/N_0) R_i}{\sum_{k \in V} R_k (E_b/N_0) R_k} \quad (32)
\]

We will show how the power allocated at base station configuration \( j \) to serve virtual \( \text{stp} \) \( V \) (\( P_{Vj} \)) can be split into \( |V| \) components, each denoted \( P_{ij} \) to support the individual \( \text{stp} \) \( i \in V \). Let \( P_{ij} = \gamma_i P_{Vj} \) and thus \( \sum_{i \in V} P_{ij} = P_{Vj} \).

Since \( p_{Vj} \) is calculated using \( T_{L_{Vj}}^{\text{wanted}} \) where \( T_{L_{Vj}}^{\text{wanted}} \geq T_{L_{ij}} \), we have:

\[
p_{ij} \geq \gamma_i p_{Vj} \quad (33)
\]
Since $\forall i \in V$:

$$\frac{R_i}{W}(E_b/N_o)_{R_i} \geq V_{\text{min}} \quad (34)$$

we have that:

$$\gamma_i p_{V_j} \geq \frac{V_{\text{min}}}{\sum_{k \in V} \frac{R_k}{W}(E_b/N_o)_{R_k}} \cdot p_{V_j} \quad (35)$$

Since Equation 29 is satisfied, it follows from (35) that $\forall i \in V$:

$$\gamma_i p_{V_j} \geq \pi \quad (36)$$

and so Equation 5 satisfied for all $i \in V$, as required.

Now consider Equation 30 which is satisfied by definition. After multiplying Equation 30 by $\frac{R_i}{W}(E_b/N_o)_{R_i}$ and rearranging, it follows that for $stp i$:

$$\gamma_i \cdot \frac{p_{V_j}}{N + I_{V_j}(1 - \alpha) + \sum_{k \in S^* \setminus j} I_{V_k}} \geq \frac{R_i}{W}(E_b/N_o)_{R_i} \quad (37)$$

Since $I_{V_j}$ is calculated using $TL_{V_j}^{\text{unwanted}}$ where $TL_{V_j}^{\text{unwanted}} \leq TL_{ij} \forall i \in V$ we have $I_{ij} \leq I_{V_j} \forall i \in V$ we have:

$$N + I_{ij}(1 - \alpha) + \sum_{k \in S^* \setminus j} I_{ik} \leq N + I_{V_j}(1 - \alpha) + \sum_{k \in S^* \setminus j} I_{V_k} \quad (38)$$

From Equation 37, noting Equations 33 and 38, it follows that $\forall i \in V$:

$$\frac{p_{ij}}{N_j + I_{ij}(1 - \alpha) + \sum_{k \in S^* \setminus j} I_{ik}} \geq \frac{R_i}{W}(E_b/N_o)_{R_i} \quad (39)$$

and thus Equation 6 is satisfied as required. □

4.1 Specifying Virtual $stp$

It is important to note that we have freedom in selection of $stp$ to form virtual $stp$. However this selection is more accurate when the members of the virtual $stp$ are test points with similar physical characteristics. We define similarity between a pair of $stp$ based on the satisfaction of three requirements:
• similarity in the list order of the stp’s strongest serving candidate base station configurations;

• similarity in the total loss values from the candidate base station configurations;

• similarity in physical location of stp.

To achieve this let \( SS_i = (j_{i1}, j_{i2}, \ldots, j_{ik}) \) denote the strongest server list for stp \( i \), where \( j_{i1}, j_{i2}, \ldots, j_{ik} \in S_i \). Thus \( SS_i \) is the ordering of the \( k \) strongest serving candidate base station configurations in ascending order of total loss value for stp \( i \). Hence \( SS_i \) is such that \( TL_{i,j_{i1}} < TL_{i,j_{i2}} < \cdots < TL_{i,j_{ik}} \).

**Definition 4.** Let \( i \) and \( i' \) be stp. These stp are \( k \)-similar for parameters \((\delta, \epsilon)\) if and only if:

1. \( SS_i = SS_{i'} \);

2. \(|TL_{i,j_{ix}} - TL_{i',j_{ix'}}| < \delta \) for \( x = 1, \ldots, k \);

3. Physical distance between \( i \) and \( i' \) is less than \( \epsilon \).

The definition of \( k \)-similarity allows us to model the relationships between stp using a graph \( G^{k,\delta,\epsilon} = (V', E') \), where \( V' = S_{\text{stp}} \) and \( \{i, i'\} \in E' \) if and only if \( i, i' \in S_{\text{stp}} \) are \( k \)-similar for parameters \((\delta, \epsilon)\). Thus \( G^{k,\delta,\epsilon} \) determines the structure of relationships between stp.

We use the graph \( G^{k,\delta,\epsilon} \) to form virtual stp by partitioning the set \( S_{\text{stp}} \) into a number of disjoint subsets such that subgraph induced by each subset is a clique. Each subset is denoted as a virtual stp. This ensures that the set of stp which constitute a virtual stp are mutually \( k \)-similar for parameters \((\delta, \epsilon)\). This is of benefit because it means that the approximation for the total loss at the virtual stp \( V \) is similar to that required for all stp in \( V \). Consequently the resources required for the virtual stp are only marginally more than those required for the total of the individual stp in \( V \). It is of greatest computational benefit if the number of virtual stp is minimized, thus the number of decision variables is reduced.

Partitioning stp into virtual stp is exactly the minimum clique partition problem of a graph. Formally a clique partition of \( G = (V', E') \) is a partition of \( V' \) into \( m \) disjoint subsets \( V'_1, V'_2, \ldots, V'_m \) such that
subgraph induced by each $V'_i$, $1 \leq i \leq m$, is a clique. Minimum clique partition is a clique partition with minimum $m$. Since this is a well-known NP-Complete problem [7], an efficient heuristic is desirable to create virtual stp. We adopt a greedy approach based on Welsh-Powell algorithm [11].

4.2 Clique Partition Approximation for Virtual stp Identification

Clique partitioning of a graph $G$ is equivalent to vertex coloring of its complement graph $G'$ [3]. The set of vertices of each color in $G'$ constitutes a clique in $G$. As a result the minimum number of cliques in the clique partition of $G$ is equal to the minimum number of required color for coloring $G'$. The Welsh-Powell algorithm is basically a greedy sequential approximation algorithm for vertex coloring of a graph. First the vertices are sorted according to their decreasing order of degrees. Then the greedy algorithm considers the vertices in that specific order $v_1, v_2, \ldots, v_n$ and assigns to $v_i$ the smallest available color not used by $v_i$'s neighbors among $v_1, v_2, \ldots, v_{i-1}$, adding a fresh color if none available. It has been shown that the resulting greedy coloring uses at most one more than the graphs maximum degree.

We apply the Welsh–Powell algorithm on the complement graph of $G^{k, \delta, \epsilon}$ to find its coloring. The set of vertices of each color is then regarded as a virtual stp in $G^{k, \delta, \epsilon}$. Since each such virtual stp obtained is a clique in $G^{k, \delta, \epsilon}$ the stp in a virtual stp are mutually $k$-similar for parameters $(\delta, \epsilon)$.

5 Experimentation

In this section we will evaluate and compare the effect of virtual stp in the coverage optimization models defined in Sections 3.1 and 3.2. The aim is to determine the difference in coverage accuracy against the number of virtual stp used.

5.1 Test problems

Tests are conducted using several downlink instances with a rectangular service region, a number of base station configurations and a number of stps. We have considered three different service regions of $0.6 \times$
0.6 km (Figure 1), 1.5 × 1.5 km (Figures 2 and 4) and 3 × 3 km respectively.

In each test problem three unique candidate base station configurations are co-located at an individual site. At a site, the candidate base station configurations respectively have a wide-beam antenna with directions of 0, 120 and 240 degrees in the horizontal plane and a downtilt of 15 degrees in the vertical plane. Path loss has been calculated using the Hata model as described in Section 2.

For the service region 0.6 × 0.6 km there is only one site at which three candidate base station configurations are located. This is placed at the centre of the region. For the service region 1.5 × 1.5 km both uniform as well as random sites are considered. For the uniform positioning of the sites, the region is composed of 9 equal sized squares of 0.5 × 0.5 km and then the sites are placed in the centre of each square, at which three base station configurations are each co-located. For the random positions, the 9 sites are placed randomly in the service region. For the service region 3 × 3 km, the region is composed of 9 equal sized squares of 1 × 1 km and then the sites are placed in the centre of each square.

For the location of *stps*, the service regions 0.6 × 0.6 km, 1.5 × 1.5 km and 3 × 3 km are divided into 9 × 9, 16 × 16 and 21 × 21 regular grids, respectively where each grid point represents an *stp*. Consequently there are 81, 256 and 441 *stps*, respectively in the service regions 0.6 × 0.6 km, 1.5 × 1.5 km and 3 × 3 km.

Data rates requested by the *stps* are considered to be heterogeneous with four different data rates \(R_1 = 12.2\) kbps, \(R_2 = 64\) kbps, \(R_3 = 144\) kbps and \(R_4 = 384\) kbps. The proportion of *stps* requesting data rates \(R_1\), \(R_2\), \(R_3\) and \(R_4\) is 50%, 10%, 10% and 5% respectively, with 25% of *stps* are requesting pilot coverage only (and thus these *stp* are effectively *ptp* only). The data rate requested by each *stp* is allocated randomly. The positions of the *stp* along with their respective data rates and the transmission site positions have been shown in Figures 1, 2, 4 and 6. In all figures • represents *stp* requesting pilot coverage only, *, ◦, □ and ◄ represent *stp* requesting data rates 12.2 Kbps, 64 Kbps, 144 Kbps, and 384 Kbps, respectively. The details of the other parameter settings are describes in Table 1. The results are obtained by running a mixed integer programming solver (CPLEX 11.0) on a Linux PC (Intel Xeon E5345 2.33GHz) with 4GB RAM.
5.2 Evaluation

Our approach is to analyze the performance from virtual step parameter sensitivity, \( k, \epsilon \) and \( \delta \). We report a number of different measures throughout the results tables as follows.

- ‘Cplex Gap’ indicates the relative optimality gap (in %) at the time of termination of the mixed integer programme solver in the CPLEX programme. The gap is defined as \( \frac{|\text{best bound} - \text{best integer}|}{(10^{-10} + |\text{best integer}|)} \times 100 \) where ‘\text{best bound}’ is the best objective function value achievable and ‘\text{best integer}’ is the objective function value of the integer solution at the time of termination [1]. Large CPLEX gaps imply that CPLEX needs more time to converge to an optimal solution. CPLEX runs have been terminated when no solution improvement has been detected within a 6 hour period.

- The measured power reported in the result tables shows the total power allocated for at the selected base station configurations for serving downlink traffic which excludes the fixed pilot power assigned...
Figure 1: The uniform positions of transmission sites and the location of 81 stps with their respective data rates for the service region $0.6 \times 0.6$ km.

- $VSTP$ in the results table indicates the number of virtual stp that are produced given the particular combination of $k$, $\delta$, and $\epsilon$.

- Coverage for the original stp problem is reported as the percentage of stp that are covered.

- For virtual stp coverage refers to the percentage of the original stp that are covered as a consequence of the virtual stp.

- The ‘Coverage Gap’ indicates the reduction in percentage coverage with respect to the coverage of the original problem due to use of virtual stp.

- The ‘stp Gap’ indicates the difference between the number of stp and virtual stp as a percentage of the number of stp. This indicates the reduction in stp evaluation needed due to use of virtual stp.
Table 2: Coverage comparison between original problem and virtual STP problem for 0.6km × 0.6 km service region with 81 stp.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Original Problem</th>
<th></th>
<th>Virtual STP Problem</th>
<th></th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>δ</td>
<td>Coverage</td>
<td>Power</td>
<td>Time</td>
<td>Cplex Gap</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
<td>97.5308</td>
<td>1.200000</td>
<td>5.77</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>225</td>
<td>8</td>
<td>97.5308</td>
<td>1.200000</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6</td>
<td>97.5308</td>
<td>1.200000</td>
<td>5.77</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>6</td>
<td>97.5308</td>
<td>1.200000</td>
<td>5.77</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>6</td>
<td>97.5308</td>
<td>1.200000</td>
<td>5.77</td>
</tr>
</tbody>
</table>

- The results column titled ‘Loss Ratio’ indicates ‘Coverage Gap’ as a proportion of ‘stp Gap’. Ideally we are looking for parameter combination that leads to high ‘stp Gap’ and low ‘Coverage Gap’ which indicates that we are able to obtain near accurate estimates of coverage while evaluating fewer stp. Thus a low value for ‘Loss Ratio’ is preferable.

5.3 Coverage comparison for the 0.6 km × 0.6 km problem

Table 2 shows the coverage results obtained when selecting a given number base station configurations (Section 3.1) for a range of k, ε and δ values. Note that for this small problem instances CPLEX converges to optimal solution for each case (CplexGap = 0). This is a small test problem and run times are very small.

First 3 rows of results in Table 2 show the effect of δ on coverage when other two parameters k and ε are fixed. Note that the number of virtual stp (VSTP) is more if δ is smaller. This is because for smaller δ values, one virtual stp can include only a very few number of individual stps and in many cases, the individual stp alone is the virtual stp. On the other hand, smaller δ represents more accurate formulation of the virtual test problem leading to less approximation on the resource requirement which finally leading to more coverage.

Rows 4-6 in Table 2 show the effect of k on coverage when other two parameters δ and ε are fixed. The number of virtual stp (VSTP) is more if k is higher. As for δ, higher k values mean that one virtual stp can include only a very few number of individual stp’s.
Rows 7-9 in Table 2 shows the effect of $\epsilon$ on coverage when other two parameters $\delta$ and $k$ are fixed. Here there is a greater effect when applying virtual $stp$. There is a reduction of 19.75% to 23.45% in the number of $stp$ when the virtual $stp$ are applied. This results in a coverage reduction of 5.06% to 12.65% in the model. For increased complexity we consider the effects in the 1.5 km × 1.5 km model.

5.4 Coverage comparison for the 1.5 km × 1.5 km problem

![Graph showing coverage comparison for 1.5 km × 1.5 km problem]

Figure 2: The uniform positions of transmission sites and the location of 256 stp with their respective data rates for the service region 1.5 × 1.5 KM

For the regular positioning of the locations at which candidate base station configurations occur, Table 3 shows the optimized coverage obtained when solving the problem formulation for selection of base-station configurations (Section 3.1) for a range of $k$, $\epsilon$ and $\delta$ values. The 1.5 × 1.5 km problems are sufficiently large to impede identifying an optimal solution within the termination criteria. However reasonably close approximations are identified. An example solution for the case with 9 transmitters is shown in Figure 3, where 98.0468% coverage is achieved.
Table 3: Coverage comparison between original problem and virtual STP problem for uniform site positioning in the 1.5km × 1.5km region with 256 stp.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Original Problem Coverage</th>
<th>Original Problem Total Power</th>
<th>STP Coverage</th>
<th>STP Power</th>
<th>Coverage Gap</th>
<th>STP Gap</th>
<th>Coverage (STP)</th>
<th>STP (Gap)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>600</td>
<td>6</td>
<td>91.0156</td>
<td>2.626157</td>
<td>270274.40</td>
<td>8.87</td>
<td>213</td>
<td>82.1218</td>
<td>2.609204</td>
</tr>
<tr>
<td>5</td>
<td>500</td>
<td>6</td>
<td>91.0156</td>
<td>2.626157</td>
<td>270274.40</td>
<td>8.87</td>
<td>204</td>
<td>80.0781</td>
<td>2.613864</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>6</td>
<td>91.0156</td>
<td>2.626157</td>
<td>270274.40</td>
<td>8.87</td>
<td>204</td>
<td>80.0781</td>
<td>2.613864</td>
</tr>
</tbody>
</table>

The effect of each of the parameters on the coverage are very similar to that explained earlier for the case of 0.6 km × 0.6 km problem. The use of virtual stp leads to a maximum of a 20.31% reduction in the number of stp forming the problem. At the same time, the maximum reduction coverage due to the use of virtual stp is at most 12.02%. Note that in all cases, the stp gap is always greater than the coverage gap.

For non-regular candidate locations in the 1.5 × 1.5 km region, the placement shown in Figure 4 has been taken. The site clustering represents the varying opportunities that might be available for candidate site placement.

Table 4 shows the optimal coverage obtained by solving the problem formulation for selection of base-station configurations (Section 3.1) using a range of k, ε and δ values. Despite a very different set of candidate site locations, the trends in terms of stp gap and coverage gap are very similar (in order of magnitude) to those for the uniform case shown in Table 3. An example solution for the case of random placement with 9 transmitters is shown in Figure 5, where 93.3959% coverage is achieved.

5.5 Maximum coverage from chosen sites

In this section we evaluate virtual stp by considering optimization of coverage for a given set of base station configurations (Section 3.2). We use the 3 × 3 km service region with base station configurations as given in Figure 6.

Table 5 shows the results. The effect of each of the parameters on the coverage are consistent with
the previous experiments where selection of candidate base station configurations was also performed. The results obtained when undertaking optimal coverage determination show relatively low coverage gap and substantial stp gap. In particular, loss in coverage accuracy is at most 4.83% while between 12.02% and 17.69% fewer stp are evaluated.

6 Conclusion

Using the similarity criteria between local test points we have introduced the notion of the virtual test point. This allows test problems to be modelled with fewer variables which reduces complexity. The effect of performing coverage optimization using virtual stp has been considered using two models. This has allowed us to determine the effect of virtual stp in terms of coverage accuracy. Across all test problems considered, the use of virtual stp results in an average 12.20% reduction for the number of test points that need to be evaluated while giving a lower bound on stp coverage by an average of 6.14%.
Figure 4: The random positions of transmission sites and the location of 256 stps with their respective data rates for the service region 1.5 × 1.5 KM

Results improve when the test problem size (and therefore complexity) increases.

These results are of practical benefit because it means that virtual stp can be used to good effect in evaluating candidate network configurations for network planning. The results demonstrate that: (i) use of virtual stp results in coverage that closely approximates the values from using all stp; (ii) there is a substantial reduction in the number of test points that need evaluation when virtual stp are employed. Point (ii) is particularly beneficial for large-scale scenarios where network design problems must be driven by meta-heuristic techniques such tabu search and genetic algorithms. Here candidate network configurations need to be repeatedly evaluated for coverage and this depends on the number of test points requiring evaluation. The results mean that larger test problems can become tractable and heuristic techniques as proposed in [12] to reduce the complexity in the evaluation of coverage many also adopt virtual stp and further gain in run-time reduction.
Figure 5: Selected transmitters and their respective serving STPs for coverage 93.3593% with 9 transmitters

Table 4: Coverage comparison between original problem and virtual STP problem for randomized site positioning in the 1.5km × 1.5km region.
Figure 6: The given positions of transmission sites and the location of 441 stps with their respective data rates for the service region $3 \times 3$ KM

Table 5: Optimal result for the given set of transmitters on $3 \times 3$ km problem
References


