

# Inverse Problems Network Meeting 1

Monday, 24<sup>th</sup> April 2017 - Tuesday, 25<sup>th</sup> April 2017

WX/3.14, Queens Buildings, Cardiff University

## Abstracts of Talks

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### **DISJOINT SPARSITY FOR SIGNAL SEPARATION AND APPLICATIONS TO QUANTITATIVE PHOTOACOUSTIC TOMOGRAPHY**

Giovanni Alberti  
University of Genoa

This is joint work with H Ammari. The main focus of this talk is the reconstruction of the signals  $f$  and  $g_i$ ,  $i = 1, \dots, N$ , from the knowledge of their sums  $h_i = f + g_i$ , under the assumption that  $f$  and the  $g_i$ s can be sparsely represented with respect to two different dictionaries  $A_f$  and  $A_g$ . This generalises the well-known ‘morphological component analysis’ to a multi-measurement setting. The main result states that  $f$  and the  $g_i$ s can be uniquely and stably reconstructed by finding sparse representations of  $h_i$  for every  $i$  with respect to the concatenated dictionary  $[A_f, A_g]$ , provided that enough incoherent measurements  $g_i$ s are available. The incoherence is measured in terms of their mutual disjoint sparsity.

This method finds applications in the reconstruction procedures of several hybrid imaging inverse problems, where internal data are measured. These measurements usually consist of the main unknown multiplied by other unknown quantities, and so the disjoint sparsity approach can be directly applied. In this case, the feature that distinguishes the two parts is the different level of smoothness. As an example, I will show how to apply the method to the reconstruction in quantitative photoacoustic tomography, also in the case when the Grüneisen parameter, the optical absorption and the diffusion coefficient are all unknown.

### **DYNAMICAL PHOTOACOUSTIC IMAGING**

Marta Betcke  
University College London

In photoacoustic imaging, as in many high resolution modalities, the major bottle neck is the acquisition time of finely spatially sampled data. In particular, imaging of dynamical processes results in incomplete data. In this talk we are going to discuss variational methods for photoacoustic imaging and their particular realisation in the framework of k-space pseudo-spectral methods. Our framework is general enough to accommodate different approaches to acceleration such as subsampling or compressed sensing which allow us to collect considerably fewer measurements while maintaining good quality of the reconstruction through suitable regularisation.

# ON FOUNDATIONAL COMPUTATIONAL BARRIERS IN $l^1$ AND TOTAL VARIATION REGULARISATION IN INVERSE PROBLEMS

Anders Hansen  
Cambridge University

The use of regularisation techniques such as  $l^1$  and Total Variation in Basis Pursuit and Lasso for inverse problems has been a great success over the last decades. In this talk we will discuss universal boundaries regarding the existence of algorithms for solving these problems. For example we have the following paradox: it is impossible to design algorithms to solve these general problems accurately when given inaccurate input data, even when the inaccuracies can be made arbitrarily small. As a simple number such as  $\sqrt{2}$  never comes with an exact numerical representation, inaccurate data input is a daily encounter. The impossibility result implies that for any algorithm designed to solve these problems there will be cases where the algorithm fails in the following way: For fixed dimensions and any small accuracy parameter  $\epsilon > 0$ , one can choose an arbitrary large time  $T$  and find an input such that the algorithm will run for longer than  $T$  and still not have reached  $\epsilon$  accuracy. Moreover, it is impossible to determine when the algorithm should halt to achieve an  $\epsilon$  accurate solution, and hence the algorithm will never be able to produce an output where one knows that the output is at least  $\epsilon$  accurate. The largest  $\epsilon$  for which this failure happens is called the Breakdown- $\epsilon$ . For Basis Pursuit and Lasso, the Breakdown- $\epsilon > \frac{1}{3}$  even when the absolute value of the input is bounded by one and is well conditioned.

The paradox opens up for a new classification theory to determine the boundaries of what computers can achieve in regularisation and inverse problems, and to explain why empirically many modern algorithms for solving regularisation problem in real-world scenarios perform very well. We will discuss positive classification results showing that sparse problems can be computed accurately. However, this is delicate; e.g. given standard assumptions from sparse recovery, there are algorithms that can compute a solution to Basis Pursuit accurately, however, this is impossible for Lasso and Basis Pursuit with noise parameter  $\delta > 0$ . However, one can compute a solution accurately up to the Breakdown- $\epsilon$  that tends to zero when  $\delta$  tends to zero, and coincides with the error bound provided in the theory of sparse recovery. This helps explaining the success of many modern algorithms applied in numerous real-world scenarios, and also explains the cases where algorithms will fail and why.

# VARIATIONAL GAUSSIAN APPROXIMATION FOR POISSON DATA

Bangti Jin

University College London

The Poisson model is frequently employed to describe count data, but in a Bayesian context it leads to a numerically intractable posterior probability distribution. In this talk, I will discuss a variational Gaussian approximation for approximating the posterior distribution arising from the Poisson model with a Gaussian prior. This is achieved by seeking an optimal Gaussian distribution minimizing the Kullback-Leibler divergence, or equivalently maximizing the lower bound for the model evidence. We shall derive an explicit expression for the lower bound, and establish the existence and uniqueness of the optimal Gaussian approximation. The lower bound functional can be viewed as a variant of classical Tikhonov regularization, but penalizing also the covariance of the estimate, instead of only the mean. We develop an efficient alternating direction maximization algorithm for solving the problem, and discuss the strategies for reducing the computational complexity via low rank structure of the forward operator and the sparsity of the covariance. Further, as an illustration of the use of the lower bound, we discuss the hierarchical Bayesian modeling for automatically selecting the hyperparameter in the prior distribution, and propose a monotonically convergent algorithm for determining the hyperparameter. We present extensive numerical experiments to illustrate the Gaussian approximation and the algorithms.

## DETECTION OF MULTIPLE DAMAGE IN STRUCTURES USING NATURAL FREQUENCY DEGRADATIONS

David Kennedy

Cardiff University

Non-destructive measurement of natural frequencies provides a simple means of detecting the location and severity of damage in a structure through changes in its eigenvalues. Further information is provided by changes in vibration modes (i.e. the eigenvectors), but this requires a scan of the whole structure which may be impractical for in situ assessment. Eigenvalue changes for a single occurrence of damage give a "damage signature" which can be expressed analytically and normalised so that its components are functions of the damage location but are independent of its severity. Thus, by using measured natural frequencies from the structure before and after the occurrence of damage, the location is found by solving an inverse problem, and then the severity is determined by comparison with the unnormalised damage signature. Noise in measurements can be accounted for by using a least squares search and/or interval arithmetic. If there are  $n$  occurrences of damage, the inverse problem entails a search for  $n$  damage locations and  $n$  parameters representing the damage severity. The degradations in any  $n$  of the natural frequencies are used to isolate the severity parameters, and then the remaining degradations are used to identify the locations. Preliminary results will be presented for the detection of multiple cracks in beams and frame structures.

# LARGE-SCALE INVERSE PROBLEMS IN SEISMOLOGY

Tristan van Leeuwen  
Utrecht University

Detailed maps of subsurface rock properties can be estimated from seismic observations by solving a non-linear inverse problem. Applications of this technique include archeology, civil engineering, oil and gas exploration and earth-sciences. The inverse problem can be cast as a non-linear optimization problem that aims to fit observed to modeled data. To ensure a unique solution, regularization is required. Even when the solution is unique, it may be hard to find due to the non-linearity of the problem. In this talk I will give an overview of some recent developments that address these issues. These include advanced regularization for geometric inverse problems and constraint-relaxation to convexify the optimization problem.