

## **Inverse Problems Network Meeting 2**

Thursday, 23<sup>rd</sup> November 2017 - Friday, 24<sup>th</sup> November 2017

Isaac Newton Institute, Cambridge

### **Abstracts of Talks**

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#### **COMBINING RECONSTRUCTION AND CLASSIFICATION WITH SOME APPLICATIONS IN NON-LINEAR INVERSE PROBLEMS**

Prof S Arridge  
University College London

Analysis of medical images often proceeds by applying some of the well-known and successful segmentation or labelling approaches from machine vision, under the assumption that the images have been robustly reconstructed. In the case of ill-posed and especially non-linear inverse problems the reconstruction process can lead to considerable artefact in the images which is not always removed by standard regularisation techniques. In this talk we present an approach that applies classification algorithms within iterative reconstruction algorithms. Results are presented for some non-linear parameter identification problems.

#### **ACCELERATION OF PRIMAL-DUAL MINIMIZATION: DETERMINISTIC VS STOCHASTIC.**

Prof A Chambolle  
CMAP Ecole Polytechnique

In this work (joint with M. Ehrhardt, C.B. Schönlieb (Cambridge) and P. Richtarik (Kaust)) I will describe the basic techniques to show convergence and accelerated convergence of primal-dual type algorithms. I will then introduce a stochastic variant of a primal-dual algorithm and show that it has essentially the same structure as its deterministic counterpart and can be accelerated in the same way, yielding, in fact, better convergence rates.

## **FAST DATA DRIVEN COMPRESSED SENSING (WITH APPLICATION TO COMPRESSED QUANTITATIVE MRI)**

Prof M Davies  
University of Edinburgh

We consider the problem of a class of compressed sensing with a data driven signal model. We show that fast reconstruction can be achieved through an inexact iterated projected gradient algorithm along with a cover tree data structure to enable fast nearest neighbor searches. We present both theoretical and numerical results showing that significant computational savings are possible through the use of inexact projections and a fast approximate nearest neighbor search. We then apply this to a novel form of MR imaging called Magnetic Resonance Fingerprinting (MRF) that enables direct estimation of the T1, T2 and proton density parameter maps for a patient through an undersampled k-space sampling and BLIP, a gradient projection algorithm that enforces the MR Bloch dynamics.

## **NOVEL INVERSION METHODS FOR MAGNETOENCEPHALOGRAPHY(MEG)AND FOR SINGLE PHOTON EMISSION COMPUTERISED TOMOGRAPHY(SPECT)**

Prof A S Fokas  
Cambridge University

Following earlier results of the author where the radial component of the magnetic field was expressed in terms of the "visible" component of the cortical neuronal current,an analogous formula will be presented for the magnetic field itself.Furthermore,a possible approach for determining the visible component of the current from MEG measurements will be discussed.

Comparisons of the implementation of a novel numerical technique for the inversion problem in PET versus the standard techniques of FBP and OSEM were presented in two recent papers in Medical Physics.Analogous comparisons will be presented for SPECT.

## **MULTI-DIMENSIONAL SPARSE SUPER-RESOLUTION**

Dr C P Poon  
University of Cambridge

This talk is concerned with the super-resolution problem for positive spikes in arbitrary dimensions. More precisely, I will discuss the issue of support recovery for the so-called BLASSO method. While super-resolution is of paramount importance in overcoming the limitations of many imaging devices, its theoretical analysis is still lacking beyond the 1-dimensional case. The reason is that in the 2-dimensional case and beyond, the relative positions of the spikes enter the picture, and one needs to account for these different geometrical configurations. After presenting an algorithmic description of the limit of the associated dual problems as the spikes cluster around a given point, I will present a detailed analysis of the support stability and super-resolution effect in the case of a pair of spikes. This is joint work with Gabriel Peyre.

# **REGULARIZATION THEORY IN INFINITE DIMENSIONS AND APPLICATIONS TO PHOTOACOUSTICS IMAGING**

Prof O Scherzer  
University Vienna

In this talk we review recent results on regularization theory of variational regularization methods in infinite dimensions. In particular we are reviewing new convergence rates results for solving Inverse Problems in Hilbert Spaces. Particular emphasize will be given to analyze and interpret the abstract results for photoacoustic imaging and inversion of the spherical mean operator.

## **MODEL-BASED LEARNING IN IMAGING**

Dr C Schonlieb  
Cambridge

One of the most successful approaches to solve inverse problems in imaging is to cast the problem as a variational model. The key to the success of the variational approach is to define the variational energy such that its minimiser reflects the structural properties of the imaging problem in terms of regularisation and data consistency.

Variational models constitute mathematically rigorous inversion models with stability and approximation guarantees as well as a control on qualitative and physical properties of the solution. On the negative side, these methods are rigid in a sense that they can be adapted to data only to a certain extent.

Hence researchers started to apply machine learning techniques to “learn” more expressible variational models. The basic principle is to consider a bilevel optimization problem, where the variational model appears as the lower-level problem and the higher-level problem is the minimization over a loss function that measures the reconstruction error for the solution of the variational model. In this talk we discuss bilevel optimisation, its analysis and numerical treatment, and show applications to regularisation learning, learning of noise models and of sampling patterns in MRI.

This talk includes joint work with M. Benning, L. Calatroni, C. Chung, J. C. De Los Reyes, M. Ehrhardt, G. Maierhofer, F. Sherry, and T. Valkonen.

# EXACT DISCRETIZATION AND SOLUTION OF CONTINUOUS-WORLD INVERSE PROBLEMS

Prof M Unser

École Polytechnique Fédérale de Lausanne

Regularization is a classical technique for solving ill-posed inverse problems, but it is usually imposed in the discrete domain. In this talk, we consider a continuous-world scenario where an unknown function  $f$  is probed with a finite number of linear functionals (forward imaging model) corrupted by measurement noise. The non-conventional aspect is our use of a continuous-domain regularization involving the  $L_p$  norm of  $Lf$  where  $L$  is a suitable differential operator. We present two representer theorems that provide the parametric form of the solution(s) of the reconstruction problem with Tikhonov ( $p = 2$ ) vs. total-variation ( $p = 1$ ) regularization. Remarkably, the solutions of both problems are (generalized) splines with the knots being fixed for  $p = 2$  and adaptive (and fewer) in the total variation scenario. These findings suggest an exact discretization of the problem that can then be solved using finite-dimensional minimization techniques. We illustrate the theory with examples of signal reconstruction that confirm the sparsifying (resp., smoothing) effect of total variation vs. Tikhonov regularization.

Joint work with Harshit Gupta and Dr. Julien Fageot