Inverse Problems Network Meeting 5

Thursday, 23rd May 2019 - Friday, 24th May 2019

University of Kent

Abstracts of Talks

UNIQUENESS THEOREMS IN INVERSE SPECTRAL AND SCATTERING THEORY

Prof. C Bennewitz
Lund University

I shall discuss some uniqueness theorems in inverse theory, starting with the classical Borg-Marčenko theorem and moving on to more general results due to myself, in most cases in collaboration with Malcolm Brown and Rudi Weikard. If time allows I will also mention some results on inverse one-dimensional scattering.

INVERSE PROBLEMS IN MODELLING CREEP AND RELAXATION OF POLYMERIC MATERIALS

Professor A R Davies
Cardiff University

Most materials encountered in everyday life are viscoelastic in nature. Examples include plastics, composites, foods, biological fluids, oils, paints and gels, all of which are polymeric or biopolymeric. Both viscous and elastic properties pertain in these materials, and when subjected to an applied force the resulting deformation is a combination of viscous response and elastic response. There exist many constitutive models which describe the time-dependent relationships between stress and strain in deformations and flows of such materials. These models can be of differential or integral type, and can portray various degrees of complexity in the materials represented.
In all models, however complex, there is present a key material function, $\mathcal{G}(t)$ which is the relaxation modulus of the material. $\mathcal{G}(t)$ is normally modelled as the Laplace transform of a positive measure, and as such is an example of a completely monotonic function. Another important material function is the creep compliance $\mathcal{J}(t)$, the derivative of which is also completely monotonic. $\mathcal{G}(t)$ and $\mathcal{J}(t)$, are related by the Volterra integral equation

$$\int_0^t \mathcal{G}(t-t')\mathcal{J}(t')dt' = t,\ t \geq 0$$

In principle, $\mathcal{G}$ can be found from a relaxation experiment, while $\mathcal{J}$ can be found from a creep experiment, although it is usually not possible to perform both experiments on the same material. In this talk it is shown how the two functions can be determined simultaneously from a single experiment. The analysis highlights a system of embedded inverse problems.

CONTINUED FRACTION EXPANSIONS AND GENERALIZED INDEFINITE STRINGS

Dr J Eckhardt
Loughborough University/University of Vienna

Stieltjes continued fraction expansions play a decisive role in the solution of the inverse spectral problem for Krein strings. Certain continued fractions of a modified form correspond in the same way to generalized indefinite strings. I will discuss under which conditions Herglotz-Nevanlinna functions allow such an expansion and use this to solve the inverse spectral problem for generalized indefinite strings with coefficients supported on a discrete set. The results are related to the Hamburger moment problem and multi-soliton solutions of particular nonlinear wave equations.

INVERSE PROBLEMS FOR CANONICAL SYSTEMS

Dr M Langer
University of Strathclyde

For 2×2 canonical systems a Weyl coefficient can be defined in a similar way as for Sturm–Liouville equations, which is a Nevanlinna function, i.e. it maps the upper half-plane into itself. It was shown by Louis de Branges that there is one-to-one correspondence between all canonical systems (up to reparameterisation) and all Nevanlinna functions. It is the intuition that the local behaviour of the Hamiltonian at the left endpoint is related to the behaviour of the Weyl coefficient at infinity. In this talk I will present various results confirming this intuition.

WHAT PDE INVERSE PROBLEMS CAN TELL US ABOUT CLASSICAL 1D INVERSE SPECTRAL PROBLEMS, AND VICE-VERSA

Prof. M Marletta
Cardiff University

This talk is based on the article 'Uniqueness from discrete data in an inverse spectral problem for a pencil of ordinary differential operators', by Brown, Marletta and Symons, which appeared in J LMS in 2016.

We examine the reconstruction of the Titchmarsh-Weyl m-function for a class of singular 1D operator pencils, from values at quadratically spaced points. These results substantially generalise 2009 work of Rybkin and Tuan, which was based on Barry Simon's A-amplitude. Our methods are different because the A-amplitude approach no longer applies to pencil problems.

Both our results, and those of Rybkin and Tuan, eventually lead to the unique recovery of coefficients in the differential expressions from point-values of the m-function at quadratically spaced points. We observe that by constructing suitable Calderon toy problems one can interpret the 1D ODE uniqueness of Rybkin-Tuan in terms of well known uniqueness results for the Schrodinger equation in 2D. Our own result, in the reverse direction, implies a uniqueness result for 2D problems with a Berry-Dennis singularity on the boundary.

INVERSE PROBLEM FOR A SEMI-LINEAR ELLIPTIC EQUATION

Dr L Oksanen
University College London

We consider the Dirichlet-to-Neumann map, defined in a suitable sense, for the equation $-\Delta u + V(x, u) = 0$ on a compact Riemannian manifold with boundary. We show that, under certain geometrical assumptions, the Dirichlet-to-Neumann map determines V for a large class of non-linearities. The proof is constructive and is based on a multiple-fold linearization of the semi-linear equation near complex geometric optics solutions for the linearized operator, and the resulting non-linear interactions. This approach allows us to reduce the inverse problem boundary value problem to the purely geometric problem to invert a family of weighted ray transforms. This is a joint work with Ali Feizmohammadi.

NUMERICAL INVERSE SCATTERING FOR INTEGRABLE SYSTEMS, REVISITED

Dr S Olver
Imperial College London

Beginning in 2011, Tom Trogdon (U. Washington) and I began studying the numerical realisation of the inverse scattering transform used in integrable systems, building on my numerical method for solving matrix-valued Riemann–Hilbert problems with junction points. Since then, there have been significant advances on both the software side—with the development of the Julia programming language—and on the mathematical end—with the development of banded spectral methods for singular integral equations, with R. Mikael Slevinsky (U. Manitoba), as implemented in the Julia package SingularIntegralEquations.jl. This talk investigates the ongoing implementation of RiemannHilbert.jl, a Julia package for solving Riemann–Hilbert problems that builds on SingularIntegralEquations.jl, and its application to integrable systems such as KdV.

GEOMETRIC AND OBSTACLE SCATTERING AT LOW ENERGY

Dr A Waters
University of Groningen

We consider scattering theory of the Laplace Beltrami operator on differential forms on a Riemannian manifold that is Euclidean at infinity. The manifold may have several boundary components caused by obstacles at which relative boundary conditions are imposed. Scattering takes place because of the presence of these obstacles and possible non-trivial topology and geometry. Unlike in the case of functions eigenvalues generally exist at the bottom of the continuous spectrum and the eigenforms represent cohomology classes. These eigenforms appear in the expansion of the resolvent, the scattering matrix, and the spectral measure for small spectral parameter λ. We will show that certain cohomology classes can then also be represented as limits of generalised eigenfunctions, and we give formulae for the expansion of the generalised eigenfunctions in any dimension $d > 1$ near 0. In even dimensions the expansion is in terms of λ and log λ. The theory of Hahn holomorphic functions is used to describe these expansions effectively.
This is joint work with Alexander Strohmaier.