Inverse Problems Network Meeting 6

Thursday, 12^{th} December 2019 - Friday, 13^{th} December 2019

University of Manchester

Abstracts of Talks

NOVEL COMPUTATIONAL METHODS FOR MEDICAL IMAGING

Prof Thanasis Fokas
University of Cambridge

First, a general introduction of the importance of functional medical imaging will be presented, and then certain novel computational algorithms will be discussed. In particular, it will be shown that the combination of the so-called spline reconstruction technique and of a new analytic inversion of the attenuated Radon transform provide an efficient alternative to existing commercial algorithms for SPECT. In addition, the definitive answer to the non-uniqueness question for both EEG and MEG will be provided, and effective algorithms for extracting maximum information from EEG and MEG data about the 'visible' part of the neuronal current that has generated these data will be described.

STABILITY AND RECONSTRUCTION IN INVERSE PROBLEMS

Dr Romina Gaburro
University of Limerick

We discuss the issues of stability and reconstruction in inverse problems with a particular focus on Calderón's inverse conductivity problem. Given the ill-posedeness of this inverse problem, it is necessary to reformulate the issue of stability (the continuous dependence of the conductivity from the data) within the theory of ill-posed problems. In this context by stability we mean the continuous dependence of the conductivity from the data when additional *a-priori* information on the unknown conductivity is available. Niculae Mandache, while visiting Slava Kurylev at Loughborough University, showed by example that, using as *a-priori* information on the conductivity any kind of bound on any finite number of its derivatives, the best possible stability is the logarithmic type. It seems therefore that in order to gain a better type of stability, one should introduce *a-priori* information of a different kind and that is physically meaningful to the application in mind. As is well known the matter of stability is of fundamental importance in the reliability of any reconstruction procedure since, in practice, the data of the problem will be affected by errors.

RESONATORS AND ANTENNAS FROM SUB-RIEMANNIAN GEOMETRY

Prof Alan Greenleaf
University of Rochester

I will describe work with Slava, some of it published and some not, on designing two- and three-dimensional arrays of metamaterials that behave as if they are higher dimensional in various ways. Based on ideas from sub-Riemannian geometry, these have higher counts of resonant frequencies and/or stronger focussing of waves than one would expect from their physical dimension, a phenomenon we call superdimensionality. Examples are based on Grushin-type operators and the sub-Laplacian on the Heisenberg group.

This is joint work of Slava with Henrik Kettunen, Matti Lassas, Gunther Uhlmann and myself.

MINIMIZING GEODESIC EXTENSIONS AND ROUGH REGULARITY OF DISTANCE COORDINATES

Prof Sergei Ivanov
Steklov Institute of Mathematics

In some problems related to Riemannian metrics, it is convenient to use point distances as local coordinates. However one has to be careful about regularity of such coordinates. This usually means that one has to stay away from cut points and require extra regularity of the metric. I will speak about Alexandrov-geometry approach to the issue, where the distance coordinates need not be regular but they are nevertheless good enough to reconstruct the metric. The key ingredient is a minimizing geodesic extension lemma: if a geodesic segment has a minimizing extension beyond one end, then it has a minimizing extension of controlled length beyound the other end. This can be used for example for reconstruction of metrics from their distance or distance difference representations.

LARGE INVERSE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS – A MEMORIAL LECTURE FOR SLAVA KURYLEV

Prof Matti Lassas
Helsinki

Discuss Slava's work and his life

INVERSE PROBLEMS FOR NON-LINEAR PDES

Dr Lauri Oksanen
University College London

One of Slava Kurylev's last contributions to the field of inverse problems was a new approach to solve coefficient determination problems for non-linear partial differential equations. Together with M. Lassas and G. Uhlmann, he showed how to recover the conformal class of a Lorentzian manifold given a local source-to-solution map associated to a wave equation with a quadratic non-linearity on the manifold. Strikingly, their result covers a large class of geometric settings for which the corresponding problem for the linear wave equation is open. I will discuss some recent results inspired by their approach.

CONSISTENT INVERSION OF NOISY NON-ABELIAN X-RAY TRANSFORMS

Prof Gabriel Paternain
University of Cambridge

I will discuss the problem of how to reconstruct a skew-hermitian field from its noisy scattering data (with applications to Polarimetric Neutron Tomography). In particular, I will address the question of when posterior distributions arising from Gaussian priors, and their posterior means consistently solve this non-linear statistical inverse problem in the (frequentist) large sample size scenario.

This is joint work with F. Monard and R. Nickl.

THE RESOLVENT OF THE NAVIER EQUATIONS IN ELASTICITY.

Prof Alberto Ruiz
Universidad Autónoma de Madrid

We will extend the resolvent estimates of Laplace operator for the Navier System. We will see applications of the resolvent to inverse problems and the problems that remain to be solved. It is a joint work with J.A. Barceló, J. Wright and M Vilela.

GLOBAL HYPERBOLIC PROPAGATORS IN CURVED SPACE

Prof Dmitri Vassiliev
University College London

Consider a hyperbolic linear partial differential equation (PDE) or system of PDEs. The propagator is the linear operator mapping initial conditions (Cauchy data) to the solution of the hyperbolic equation or system.

Our aim is to construct explicitly, modulo smooth terms, propagators for physically meaningful PDEs and systems of PDEs on manifolds without boundary, and to do this in a global (i.e. as a single oscillatory integral) and invariant (under changes of local coordinates and any gauge transformations that may be present) fashion. Here by “explicitly” we mean reducing the PDE problem to integration of ordinary differential equations.

The crucial element in our global construction is the use of a complex-valued, as opposed to real-valued, phase function - an idea proposed in earlier publications by the speaker and co-authors. It is known that one cannot achieve a construction global in time using a real-valued phase function due to topological obstructions (caustics), however it turns out that the use of a complex-valued phase function allows one to circumvent these topological obstructions.

The three main mathematical models to be discussed in the talk are the wave equation, the massless Dirac equation and Maxwell’s equations.

The construction discussed in the talk is a development of that described in arXiv:1902.06982.