Meeting on Modern Aspects of Analysis and Scientific Computing

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Abstracts of Talks

MULTIPLE ORTHOGONAL POLYNOMIALS AND SOME OF THEIR APPLICATIONS

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Multiple orthogonal polynomials are polynomials in one variable that satisfy orthogonality conditions with respect to several measures. I will briefly give some general properties of these polynomials (recurrence relation, zeros, etc.). These polynomials have recently appeared in many applications, such as number theory, random matrices, non-intersecting random paths, integrable systems, etc. I will speak about two applications. The first is an extension of the Gauss quadrature formula for integrating a function, to simultaneous quadrature where one wants to integrate one function with respect to several measures. The second application is the construction of multiwavelets by means of polynomials, which uses Legendre polynomials and multiple orthogonal polynomials known as Legendre-Angelesco polynomials.

QUASI MONTE-CARLO METHODS FOR HIGH-DIMENSIONAL INTEGRALS AND EXPECTATIONS

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We give an overview of modern quasi-Monte Carlo (QMC) methods to tackle high-dimensional integrals. Dimensions ranging from 3 up to thousands can be handled easily using quasi-Monte Carlo methods. We show modern QMC methods can show algebraic convergence of $O(N^{-1})$, $O(N^{-2})$ and $O(N^{-3})$ for practical problems, where even the constant can be independent of the number of dimensions. We finish by giving some pointers to software to construct (polynomial) lattice rules and to point generators for these point sets.

ON POLYNOMIAL EXPRESSIONS FOR HIGHER-DIMENSIONAL FIBONACCI SEQUENCES, FAULHABER SUMS, AND SOME THEOREMS OF FERMAT

Matthew Lettington
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Recently it has been shown that there exist families of polynomials which lend themselves to expressing and generalising some results of the three number theorists Fibonacci (circa 1170-1250), Johann Faulhaber (1580-1635), and Pierre de Fermat (1607-1665).

In this talk we begin by discussing some of the key properties exhibited by these polynomial families, including the more modern concepts of orthogonality conditions, links to Chebyshev functions, differential equations and that the Mellin transforms of these polynomials satisfy a functional equation of the form $p_n(s) = \pm p_n(1-s)$.

With the definitions in place we then give an overview of some of the main results derived using these polynomials. In particular we consider how these polynomials can be used to

(i) generate higher-dimensional interlacing Fibonacci sequences from which we construct sequences of vectors in $\mathbb{Q}^m$, which converge to irrational algebraic points in $\mathbb{R}^m$,

(ii) generalise the concept of the Golden Ratio,

(iii) express Faulhaber's r-fold sums of the mth powers of the first n integers,

(iv) express some theorems of Fermat, where in particular we obtain an explicit expression for the integer quotient q, in Fermat's Little Theorem, which says that for p a prime and a an integer with hcf$(a,p) = 1$, one can write $a^{p-1} = qp + 1$.

LEARNING POSSIBILISTIC LOGIC THEORIES FROM DATA

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Possibilistic logic is a generalization of classical logic, in which formulas are associated with certainty degrees from the unit interval. At the semantic level, a possibilistic logic theory represents a possibility distribution over the set of possible worlds. While a wide range of applications have already been studied, only few authors have looked at methods for learning possibilistic logic theories from data. In this talk, I will discuss two such methods. The first method relies on the ability of possibilistic logic to compactly represent sets of default rules of the form "if A then typically B". In particular, the proposed method is aimed at learning a possibilistic logic theory from a large set of noisy default rules. Among others, this allows us to use crowdsourcing methods for learning consistent domain theories. For the second method, the input consists of a set of examples that are represented using standard feature vectors. The proposed method learns a possibilistic logic theory that models a probability distribution that is estimated from these examples. In this case, the machinery of possibilistic logic is used for probabilistic inference. The learned possibilistic logic theories can be used for maximum a posteriori inference, as well for evaluating marginal and conditional probabilties. In this way, possibilistic logic can be used as the basis for interpretable machine learning methods.

MORTON ORDERING OF 2D ARRAYS FOR PARALLELISM AND EFFICIENT ACCESS TO HIERARCHICAL MEMORY

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This talk describes the recursive Morton ordering that supports efficient access to hierarchical memory across a range of heterogeneous computer platforms, ranging from many-core devices, multi-core processor, clusters, and distributed environments. Programmer-level control of the memory hierarchy is also considered. A brief overview of previous research in this area is given, and algorithms that make use of Morton ordering are described. These are then used to demonstrate the efficiency of the Morton ordering approach by performance experiments on different processors. In particular, timing results are presented for matrix multiplication, Cholesky factorisation, and fast Fourier transform algorithms. The use of the Morton ordering approach leads naturally to algorithms that are recursive, and exposes parallelism at each level of recursion. Thus, the approach advocated in this talk not only provides convenient and efficient access to hierarchical memory, but also provides a basis for exploiting parallelism.

SOME SPECTRAL RESULTS FOR WAVEGUIDES

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We study a spectral problem for the Laplacian in a weighted space which is related to the propagation of electromagnetic waves in photonic crystal waveguides. The waveguide is created by introducing a linear defect into a periodic medium. The defect is infinitely extended and aligned with one of the coordinate axes. The perturbation introduces guided mode spectrum inside the band gaps of the fully periodic, unperturbed spectral problem. We use variational arguments to prove that guided mode spectrum can be created by arbitrarily small perturbations. After performing a Floquet decomposition in the axial direction of the waveguide, we study the spectrum created by the perturbation for any fixed value of the quasi-momentum. Time permitting, we will also briefly discuss extending the results to a similar problem for divergence form elliptic operators.