

# Meeting on Modern Aspects of Analysis and Scientific Computing

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## Abstract of Talk

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### ON POLYNOMIAL EXPRESSIONS FOR HIGHER-DIMENSIONAL FIBONACCI SEQUENCES, FAULHABER SUMS, AND SOME THEOREMS OF FERMAT

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Recently it has been shown that there exist families of polynomials which lend themselves to expressing and generalising some results of the three number theorists Fibonacci (circa 1170-1250), Johann Faulhaber (1580-1635), and Pierre de Fermat (1607-1665).

In this talk we begin by discussing some of the key properties exhibited by these polynomial families, including the more modern concepts of orthogonality conditions, links to Chebyshev functions, differential equations and that the Mellin transforms of these polynomials satisfy a functional equation of the form  $p_n(s) = \pm p_n(1-s)$ .

With the definitions in place we then give an overview of some of the main results derived using these polynomials. In particular we consider how these polynomials can be used to

- (i) generate higher-dimensional interlacing Fibonacci sequences from which we construct sequences of vectors in  $\mathbb{Q}^m$ , which converge to irrational algebraic points in  $\mathbb{R}^m$ ,
- (ii) generalise the concept of the Golden Ratio,
- (iii) express Faulhaber's  $r$ -fold sums of the  $m$ th powers of the first  $n$  integers,
- (iv) express some theorems of Fermat, where in particular we obtain an explicit expression for the integer quotient  $q$ , in Fermat's Little Theorem, which says that for  $p$  a prime and  $a$  an integer with  $\text{hcf}(a, p) = 1$ , one can write  $a^{p-1} = qp + 1$ .