m-Function and Related Topics Conference

Cardiff University, 19^{th}-21^{st} July 2004

Cardiff School of Computer Science

Cardiff School of Mathematics

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Cardiff School of Computer Science, Cardiff University, Cardiff CF24 3XF, Wales, UK

Cardiff School of Mathematics, Cardiff University, Cardiff CF24 4YH, Wales, UK

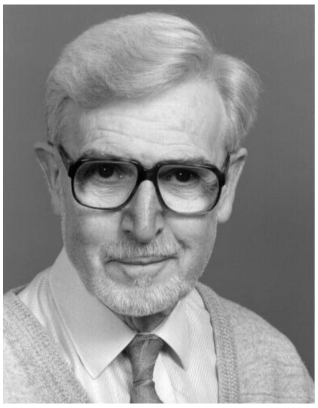
This conference has been organised by B M Brown (Cardiff), W D Evans (Cardiff), R Weikard (Birmingham, AL) and C Bennewitz (Lund). It is supported by the EPSRC, LMS and NSF.

Contents

Laudatum

B M BROWN & M S P EASTHAM & W D EVANS

The spectral theory of differential equations has been a central theme of mathematics for over 200 years and the long list of contributers includes such early names as Fourier, Sturm and Liouville. With the developing techniques of analysis in the 19th century, the foundations of the modern theory were laid by Weyl, Titchmarsh, Hartman and Wintner, Kodaira and, in Russia, by Naimark and Levitan.



Over the past 50 years Norrie Everitt has built on these foundations with his own important contributions which have also inspired widespread research by others. Of the major ideas which Norrie has initiated, mention must first be made of his work on the Weyl limit-point, limit-circle theory for higher-order (even and odd) differential equations using the notion of Gram determinants [4],[5],[7], [9].

This work naturally led on to the now familiar deficiency index problem [11] where, at an early stage, Norrie drew attention to the developments in Russia by making available the English edition of Naimark's book on linear differential operators. As a refinement of this problem, Norrie identified the concepts of the strong limit$-n$ classification and separation [6],[13].

In a further seminal paper [8], Norrie initiated the study of the asymptotic form of the Titchmarsh-Weyl $m(\lambda)$ function for large $\mid \lambda \mid$. During the following 30 years, this paper has led to many fruitful and attractive papers which are now being celebrated in the Cardiff meeting of 2004.

The long-standing Hardy, Littlewood, Polya inequality relates the norms of a function and its first two derivatives. Norrie's 1972 paper [10] placed this inequality in a new spectral setting which has spawned a large class of HELP inequalities, where the E for Everitt is now included in his honour. This work continued with, amongst others, the incisive papers [2],[12], [3] together with contributions from many fellow researchers.

Norrie has for many years been interested in the spectral theory of orthogonal polynomials and connections with higher order equations and Sobolev orthogonality. He has explored the spectral theory of these problems under both the so called right-definite and left-definite hypotheses (see for example [14]).

Norrie has always been interested in exploring new aproaches to his mathematics and was instrumental (with Zettl and Bailey) in setting up the SLEIGN2 programme which resulted in a computer code to calculate the eigenvalues of the Sturm-Liouville problem under a wide range of assumptions which include the limit-point, limit-circle cases and also the periodic problem (see for example [1]). This code is in wide use today both by mathematicians and scientists who need to explore the eigenvalue structure of their problem.

These remarks are only a partial appreciation of Norrie's wide-ranging research in spectral theory, but the papers in the present volume are a testimony to the esteem with which his work is held by his many friends and colleagues.

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Timetable of Talks

Monday

MORNING SESSION CHAIR B M BROWN, FACULTY LECTURE THEATRE

9:00am	Introductory remarks
9.05am	Opening of meeting by **J B McLeod**
9:15am	**W N Everitt**: The m-coefficient and related topics
10:15am	Coffee
11:00am	**D Hinton**: Titchmarsh-Weyl Coefficients for Odd-order Linear Hamiltonian Systems
12:00pm	**C Bennewitz**: The m-function for left definite equations
1:00pm	Lunch

AFTERNOON SESSIONS

PARALLEL SESSION I, CHAIR P KURASOV, FACULTY LECTURE THEATRE

2:00pm	**S L Clark**: Weyl-Titchmarsh theory for singular finite difference Hamiltonian systems
2:30pm	**D J Gilbert**: Riccati equations and the Titchmarsh-Weyl m-function
3:00pm	**C T Fulton**: Weyl-Titchmarch m-functions for Second-order Sturm-Liouville Problems with two singular endpoints
3:30pm	**N Gordon**: Numerical calculation and investigation of m-functions

PARALLEL SESSION II, CHAIR D HINTON, T/2.07

2:00pm	**K H Kwon**: Bochner-Krall orthogonal polynomials
2:30pm	**L L Littlejohn**: Some Applications of Left-Definite Theory for Positive Self-Adjoint Operators
3:00pm	**I S Kats (Kac)**: Spectral Theory of a String and the Pathological Birth and Death Processes
3:30pm	**F H Szafraniec**: On operators that are almost creations

PARALLEL SESSION III, CHAIR K M SCHMIDT, C/2.07

2:00pm	**P Binding**: Eigenvalue signatures
2:30pm	**H O Cordes**: Some non-elliptic Sturm-Liouville problems, and their unitary singular-integral-diagonalization
3:00pm	**V Maric**: Properties of solutions of the half-linear equation
3:30pm	**K A Mirzoev**: Solutions of ordinary ordinary differential equations belonging to $L^p(w, \mathbb{R}^+)$
4:00pm	Coffee

PARALLEL SESSION I, CHAIR C BENNEWITZ, FACULTY LECTURE THEATRE

4:30pm	**G Freiling**: The Weyl-Yurko Matrix and the Method of Spectral Mappings in the Inverse Problem Theory
5:00pm	**A A Shkalikov**: Inverse problems for the Sturm–Liouville operators with distributional potential
5:30pm	**R O Hryniv**: Inverse spectral problems for Sturm–Liouville and Dirac operators with singular potentials

PARALLEL SESSION II, CHAIR W K HAYMAN, T/2.07

4:30pm	**A Gilanyi**: On inequalities and Dinghas-type derivatives
5:00pm	**Z Páles**: Hardy-type inequalities for means
5:30pm	**M Bessenyei**: Hermite–Hadamard-type inequalities for generalized convex functions

PARALLEL SESSION III, CHAIR EB DAVIES, C/2.07

4:30pm	**J Kurzweil**: The Riemannian approach to ODEs and to integration
5:00pm	**S Yu Slavyanov**: Jost Function for Confluent Heun Equation with Nearby Singularities
5:30pm	
6:00pm	End

Tuesday

MORNING SESSION CHAIR R WEIKARD, FACULTY LECTURE THEATRE

9:15am	**B Simon**: Analogs of the m-function in the Theory of Orthogonal Polynomials on the Unit Circle
10:15am	Coffee
11:00am	**F Gesztesy**: Weyl–Titchmarsh Matrices, Trace Formulas, and Borg-Type Theorems
12:00pm	**H Langer**: The generalized Schur algorithm and inverse spectral problems
1:00pm	Lunch

AFTERNOON SESSIONS

PARALLEL SESSION I, CHAIR IW KNOWLES, FACULTY LECTURE THEATRE

2:00pm	**M Jais**: Unique identifiability of the common support of coefficients of a second order anisotropic elliptic system by the Dirichlet Neumann map
2:30pm	**R Weikard**: The m-function for a complex Jacobi matrix and an application to the inverse resonance problem
3:00pm	**P Schapotschnikow**: Numerical recovery of coefficients of the Sturm-Liouville Problem on trees
3:30pm	**Y V Kurylev**: A multidimensional Borg-Levinson theorem

PARALLEL SESSION II, CHAIR L LITTLEJOHN, T/2.07

2:00pm	**A I Kozko**: On the spectral function and regularized traces of singular differential operators
2:30pm	**A S Pechentsov**: Asymptotic behavior of the spectral function in singular Sturm-Liouville problems
3:00pm	**A G Garcia**: On the Analytic Sampling Theory (and a link with the m-function theory)
3:30pm	**A Poulkou**: Sampling And Interpolation Theories Associated With First-Order Boundary Value Problems

PARALLEL SESSION III, CHAIR D PEARSON, C/2.07

2:00pm	**M Möller**: Singular Sturm-Liouville problems whose coefficients depend rationally on the eigenvalue parameter
2:30pm	**I Sorrell**: An Asymptotic Eigenvalue Expansion for the Perturbed Harmonic Oscillator
3:00pm	**V Tkachenko**: On spectra of 1d periodic selfadjoint differential operators of order 4
3:30pm	**A Rybkin**: On the Titchmarsh-Weyl m-function associated with H^{-1} potentials

4:00pm	Coffee

PARALLEL SESSION I, CHAIR B MCLEOD, FACULTY LECTURE THEATRE

4:30pm	**C Tretter**: From self-adjoint to non-selfadjoint variational principles
5:00pm	**J S Christiansen**: On the Krein and Friedrichs extensions of a positive Jacobi operator
5:30pm	**M Muzzulini**: Spectral theoretical aspects of separation of variables

PARALLEL SESSION I, CHAIR M MARLETTA, T/2.07

4:30pm	**T Levitina**: On the computation of eigenfunctions of the finite Fourier and Hankel transforms
5:00pm	**A D Wood**: The influence of G.G. Stokes on the modern asymptotic theory of differential equations
5:30pm	**J D Pryce**: Computing Taylor series and associated Jacobians for DAEs

PARALLEL SESSION I, CHAIR M LANGER, C/2.07

4:30pm	**H Behncke**: Spectral Theory of Higher Order Differential Operators
5:00pm	**P McKeag**: Functions of the Laplace Operator
5:30pm	**B A Watson**: Equivalence of inverse Sturm-Liouville problems with boundary conditions rationally dependent on the eigenparameter

6:00pm	End

Wednesday

MORNING SESSION CHAIR W D EVANS, FACULTY LECTURE THEATRE

9:15am	**S Naboko**: Functional Model and the Spectral Structure of Nonselfadjoint Operators
10:15am	Coffee
11:00am	**B S Pavlov**: Dirichlet-to-Neumann map as a tool of analytic perturbation technique on the continuous spectrum
12:00pm	**D B Pearson**: Weyl M Operators: an Extension to Higher Dimensions of the Theory of the m-function
1:00pm	Lunch

AFTERNOON SESSION A

PARALLEL SESSION I, CHAIR B PAVLOV, FACULTY LECTURE THEATRE

2:00pm	**P Kurasov**: On inverse scattering problem for quantum graphs
2:30pm	**K Robert**: Resonance optical switch: Calculation of the eigenvalues of the reduced operator
3:00pm	**M Solomyak**: On the discrete spectrum of a family of differential operators

AFTERNOON SESSION II, CHAIR M JAIS, T/2.07

2:00pm	**A Dijksma**: Rank one perturbations at infinite coupling in Pontryagin spaces
2:30pm	**W K Hayman**: On the zeros of solutions of a functional equation
3:30pm	Coffee

AFTERNOON SESSION B, CHAIR A ZETTL, FACULTY LECTURE THEATRE

4:00pm	**M Klaus**: Remarks on the spectral asymptotics of the Titchmarsh-Weyl m-function for Dirac operators and (matrix) Schrödinger operators
4:30pm	**M Langer**: A general HELP inequality connected with symmetric operators
5:00pm	**M Plum**: Titchmarsh-Weyl-Sims Theory for Non-Selfadjoint Hamiltonian Systems
5:30pm	**E B Davies**: Semi-classical Analysis and Pseudospectra
6:00pm	End of Conference

Abstracts

Spectral Theory of Higher Order Differential Operators

H Behncke

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The nature of the absolutely continuous spectrum of differential operators of the form

$$\tau y = w^{-1} \sum_{k=0}^{n} (-1)^k (p_k y^{(k)})^{(k)} - i \sum_{j=1}^{n} (-1)^j ((q_j y^{(j)})^{(j-1)} + (q_j y^{(j-1)})^{(j)})$$

on $\mathcal{L}^2(w, (0, \infty))$ is determined, when the coefficients are almost constant in a generalized sense. It is shown that λ belongs to the absolutely continuous spectrum of multiplicity k, if there are $2k$ "bounded" and $(n-k)$ exponentially decreasing and $(n-k)$ increasing solutions of $\tau y = \lambda y$. This clearcut dichotomy also prevents the appearance of singular continuous spectrum. Extensions to odd order operators are also sketched.

– . –

The m-function for left definite equations

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The Titchmarsh-Weyl m-function is associated with the equation $-u'' + qu = lu$ considered in $L^2(0, b)$, where 0 is a regular and b possibly a singular point for the equation.

When considering $-(pu')' + qu = lwu$, in a case where w is not of fixed sign, one can not use an L^2-space with weight w as the underlying Hilbert space. However, if p and q are positive one may consider the equation in a Hilbert space with norm square $\int_0^b (p|u'|^2 + q|u|^2)$, and give a complete spectral theory. Such an equation occurs naturally in the theory of the Camassa-Holm equation, a model for shallow water waves with a bi-Hamiltonian structure.

We will describe a spectral theory for such equations, based on an analogue of the classical Titchmarsh-Weyl m-function, as well as some results in the inverse spectral theory for such equations. Some discussion of left-definite Hamiltonian systems is also given.

– . –

HERMITE–HADAMARD-TYPE INEQUALITIES FOR GENERALIZED CONVEX FUNCTIONS

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Applying Tchebychev systems, the notion of convexity can be extended in a quite natural way. The generalized convexity so obtained involves, for example, the notion of higher-order monotonicity due to Popoviciu.

Joint work with Zs. Páles

Our goal is to generalize the Hermite–Hadamard inequality for this setting, i.e., to give lower and upper estimations for the integral average of any generalized convex function with some base points of the domain. The main tool of the investigations is based on the Krein–Markov representation of moment spaces induced by Tchebychev systems.

We also investigate two important classes of generalized convexity when the base points of the Hermite–Hadamard-type inequalities can be given explicitly.

– . –

EIGENVALUE SIGNATURES

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A discussion will be given of how such signatures arise, some of their uses, and some methods for their calculation (including, of course, via the m-function).

On the Krein and Friedrichs extensions of a positive Jacobi operator

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We consider an unbounded Jacobi operator T which is bounded below by some $\varepsilon > 0$. T is defined from an infinite matrix of the form

$$\begin{pmatrix} b_0 & a_0 & 0 & 0 & \dots \\ a_0 & b_1 & a_1 & 0 & \dots \\ 0 & a_1 & b_2 & a_2 & \\ \vdots & & \ddots & \ddots & \ddots \end{pmatrix},$$

where $a_n > 0$ and $b_n \geq \varepsilon$.

Provided that T is *not* essentially selfadjoint, the positive selfadjoint extensions of T lie (in a certain sense) between two special extensions, the Krein extension and the Friedrichs extension. The Krein extension has 0 as an eigenvalue and the Friedrichs extension is bounded below by ε.

Our main result is to describe the domains of the Krein and Friedrichs extensions in terms of boundary conditions. The Krein extension is related to the solution $p = (p_n)$ to the recurrence relation

$$a_{n-1}x_{n-1} + b_n x_n + a_n x_{n+1} = 0, \quad n > 0 \tag{1}$$

with initial conditions $p_0 = 1$ and $p_1 = -b_0/a_0$. On the other hand, the Friedrichs extension is related to the minimal (or principal) solution $u = (u_n)$ to (1), i.e., the up to constant multiples unique solution such that

$$\lim_{n\to\infty} \frac{u_n}{x_n} = 0$$

for every solution $x = (x_n)$ which is linearly independent of u.

Now and then we explain the connection to the indeterminate Stieltjes moment problem.

The talk is based on joint work with B. Malcolm Brown.

WEYL-TITCHMARSH THEORY FOR SINGULAR FINITE DIFFERENCE HAMILTONIAN SYSTEMS

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We discuss the basic theory of matrix-valued Weyl-Titchmarsh m-functions and the associated Green's matrices for self-adjoint Hamiltonian finite difference systems for limit point and limit circle cases.

This is joint work done with F Gesztesy.

– . –

SOME NON-ELLIPTIC STURM-LIOUVILLE PROBLEMS, AND THEIR UNITARY SINGULAR-INTEGRAL-DIAGONALIZATION

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The spectral theory of the operator $u(x) \to xu(x) + \varepsilon((1-\partial_x^2)^{-1}u)(x)$ in $L^2(\mathbf{R})$ (with $0 \leq \varepsilon \leq 1$ a parameter) is that of the singular Sturm-Liouville problem

$$(1) \qquad (1-\partial_x)x(1+\partial_x)v + \varepsilon v = \lambda(1-\partial_x^2)v \ ,$$

self-adjoint under the (polarized) norm $\|f\|_1^2 = \|f\|^2 + \|\partial_x f\|^2$, but non-elliptic, since the highest coefficient of (1) vanishes at $x = \lambda$.

This problem is diagonalized by a unitary singular integral operator of the form

$$(2) \qquad U = a_\varepsilon + k_\varepsilon *$$

with a singular convolution $k_\varepsilon *$, where $a_\varepsilon, k_\varepsilon$ may be expressed in terms of Bessel (or Whittaker) functions.

We study a generalized such problem (of higher order) where U is a Wiener-Hopf-type singular integral operator on the half-line. We believe this problem is of importance for a cleaned theory of observables for quantum theory of the Dirac equation.

SEMI-CLASSICAL ANALYSIS AND PSEUDOSPECTRA

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We prove an approximate spectral theorem for non-self-adjoint operators and investigate its applications to second order differential operators in the semi-classical limit. This leads to the construction of a twisted FBI transform. We also investigate the connections between pseudospectra and boundary conditions in the semi-classical limit.

– . –

RANK ONE PERTURBATIONS AT INFINITE COUPLING IN PONTRYAGIN SPACES

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The lecture is based on joint work with Heinz Langer (Vienna, Austria) and Yuri Shondin (Nizhny Novgorod, Russia). We relate the operators in the operator representations of a generalized Nevanlinna function $N(z)$ and of the function $-N(z)^{-1}$ under the assumption that $z = \infty$ is the only (generalized) pole of non-positive type. The results are applied to the Q-function of S and H and the Q-function for S and H^∞, where H is a self-adjoint operator in a Pontryagin space with a cyclic element w, H^∞ is the self-adjoint relation obtained from H and w via a rank one perturbation at infinite coupling, and S is the symmetric operator given by $S = H \cap H^\infty$.

The m-coefficient and related topics

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This is a personal, and thereby selective, history of the m-coefficient as connected with the Sturm-Liouville differential equation, and the corresponding coefficient for symmetric, second-order difference equations. The account is based on my years of contact with E.C. Titchmarsh (1899-1963) in Oxford, as undergraduate student (1949-52), doctoral student (1952-54), and at the Titchmarsh seminars (1954-1963).

In places in the text I have quoted from Titchmarsh as best as memory serves, but I have been careful not to attribute definite statements to him if I am uncertain of their authority.

Here are the main items in the development of this theory and which are treated in this account:

1. The limit-point and limit-circle classification of Weyl
2. The existence of integrable-square solutions of Weyl
3. The Hellinger-Nevanlinna w-coefficient for difference equations
4. The Titchmarsh-Weyl m-coefficient.

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The Weyl-Yurko Matrix and the Method of Spectral Mappings in the Inverse Problem Theory

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We give a short review on inverse problems of spectral analysis for differential equations and systems on the half-line and on a finite interval. As the main spectral characteristics for the formulation and solution of inverse problems we use the so-called Weyl-Yurko matrix, which is one of the possible generalizations of Weyl's classical m- function. Using the concept of the Weyl-Yurko matrix and the method of spectral mappings we provide the solution of the inverse problem for non-selfadjoint differential equations and systems. We also obtain necessary and sufficient conditions for the solvability of the inverse problem.

– . –

WEYL-TITCHMARCH m-FUNCTIONS FOR SECOND-ORDER STURM-LIOUVILLE PROBLEMS WITH TWO SINGULAR ENDPOINTS

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We consider some Sturm-Liouville problems

$$-U'' + q(x)U = \lambda U, \quad x \quad \in \quad (0,\infty)$$

under the assumptions (i) q is limit point at ∞ and q is oscillatory at ∞ for $\lambda \in (\Lambda,\infty)$ and nonoscillatory at ∞ for $\lambda \in (-\infty,\Lambda)$ and (ii) q is nonoscillatory at $x = 0$ for all $\lambda \in (-\infty,\infty)$ and either limit circle or limit point. Then the continuous spectrum is contained in (Λ,∞). The spectrum is simple. We consider Weyl-Titchmarsh m-functions for such doubly singular Sturm-Liouville problems which do not fit into the usual Pick-Nevanlinna theory.

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ON THE ANALYTIC SAMPLING THEORY (AND A LINK WITH THE m-FUNCTION THEORY)

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Sampling theory deals with the reconstruction of functions through their values on an appropriate sequence of points by means of sampling expansions involving these values. We propose a sampling theory on a Hilbert space $\mathcal{H}$ of analytic functions defined as follows:

Let $\mathbb{H}$ be a complex, separable Hilbert space, let Ω be a domain in the complex plane, and suppose K is a $\mathbb{H}$-valued function on Ω. Then, $\mathcal{H}$ is the set of functions f_x, $x \in \mathbb{H}$, defined by $f_x(z) := \langle K(z), x\rangle_{\mathbb{H}}$, $z \in \Omega$.

We apply this theory to two specific examples. The first example is concerned with symmetric operators having a compact resolvent. The other example involves the application of sampling theory to the classical indeterminate Hamburger moment problem. In this example, the kernel K involves an m-function related to a Sturm-Liouville difference problem in the limit-circle case.

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WEYL–TITCHMARSH MATRICES, TRACE FORMULAS, AND BORG-TYPE THEOREMS

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Borg-type uniqueness theorems for matrix-valued Jacobi operators H and supersymmetric Dirac difference operators are proved.More precisely, assuming reflectionless matrix coefficients A, B in the self-adjoint Jacobi operator $H = AS^+ + A^-S^- + B$ (with $S^\pm$ the right/left shift operators on the lattice Z) and the spectrum of H to be a compact interval $[E_-, E_+], E_- < E_+$, we prove that A and B are certain multiples of the identity matrix. An analogous result which, however, displays a certain novel nonuniqueness feature, is proved for supersymmetric self-adjoint Dirac difference operators.

Our approach is based on Weyl-Titchmarsh matrices, (exponential) Herglotz representation theorems, and trace formulas. As a by-product of our techniques we obtain the extension of Flaschka's Borg-type result for periodic scalar Jacobi operators to the class of reflectionless matrix-valued Jacobi operators.

This is based on joint work with S. Clark and W. Renger.

On inequalities and Dinghas-type derivatives

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In his paper 'Zur Theorie der gewönlichen Differentialgleichungen' in 1966, A. Dinghas introduced the n^{th} order interval-derivative of a real function f at a point ξ by

$$D^n f(\xi) = \lim_{\substack{(\alpha,\beta)\to(\xi,\xi) \\ \alpha\le\xi\le\beta}} \left(\frac{n}{\beta-\alpha}\right)^n \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} f\left(\frac{(n-k)\alpha + k\beta}{n}\right).$$

He proved that, assuming n-times differentiability in the common sense for f at ξ, the limit above exists and it coincides with the n^{th} derivative of f. However, the class of functions differentiable in this sense is considerably wider than the family of functions differentiable in the classical setting, for example, every non-continuous additive function (the graph of which is dense in the plane) is n-times differentiable in the sense of Dinghas for all integers $n > 1$.

In the present talk we prove mean value inequalities for derivatives of the type above. As applications of our results we obtain characterization theorems for convex functions of higher order as well the localizability of some convexity properties.

Joint work with Zsolt Páles.

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Riccati equations and the Titchmarsh-Weyl m-function

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We consider some consequences of the well-known relationship between the Riccati equation and the Titchmarsh-Weyl m-function for the one-dimensional Schödinger operator.

NUMERICAL CALCULATION AND INVESTIGATION OF m-FUNCTIONS

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This talk will consider the Numerical calculation and investigation of m-functions.

For example, using forward shooting and other techniques. We will further consider the calculation and visualisation of orbits of m-functions and related values. This will include the use of Mathematica as an appropriate and convenient prototyping environment.

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ON THE ZEROS OF SOLUTIONS OF A FUNCTIONAL EQUATION

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Suppose that a, q are complex constants such that $0 <\mid q \mid< 1$ and $aq^\nu \neq 1$ when $\nu \in \mathbf{N}$. Let $f(z) = \sum_0^\infty a_n z^n$ be a solution of the functional equation

$$(a - qz)f(q^2 z) - (1 + a)f(qz) + f(z) = 0.$$

Then writing as usual $(a; q)_n = \prod_{k=0}^{n-1}(1 - aq^k)$, we have

$$a_n = a_0 q^{n^2} / \prod_{\nu=1}^{n} (1 - q^2)(1 - aq^\nu) = \frac{a_0 q^{n^2}}{(q; q)_n (aq; q)_n}.$$

In this talk I will describe a way of obtaining complete asymptotic expansions of the a_n and hence of the zeros z_n of $f(z)$. The case $a = 0$ yields a weak form of an identity from Ramanujan's lost Notebook, proved by G. E. Andrews in a different way. The method extends to more general functions $f(z)$, whose coefficients a_n are given by

$$a_n = a_0 q^{n^2} / \prod_{p=1}^{p_0} (c_p q; q)_n t_p,$$

where c_p, t_p are complex constants, such that $c_p q^k \neq 1$, $k \in \mathbf{N}$.

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Titchmarsh-Weyl Coefficients for Odd-order Linear Hamiltonian Systems

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We define the Titchmarsh-Weyl coefficient for an odd-order linear Hamiltonian system $Jy' - B(x)y = \lambda A(x)y$ in an intrinsic manner and without taking a limit of regular problems. This follows the method of our earlier work for the even order case. We consider here the case of one regular endpoint and one singular endpoint which is in the limit point case. We associate with the system and boundary conditions at the regular point a Hilbert space and an operator **B**. A difficulty is caused by the possible existence of solutions to $Jy' - B(x)y = A(x)f$ with $\|y\| = 0$ and $\|f\| \neq 0$. It is shown how the space of such y affects the definition of the Hilbert space and operator. The odd-order case causes special difficulties since there are two associated operators. The regular even order case is illustrated to show the dependence of the Hilbert space and associated self-adjoint operator on the boundary conditions.

– . –

Inverse spectral problems for Sturm–Liouville and Dirac operators with singular potentials

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We solve inverse spectral problems for Sturm–Liouville operators on $(0, 1)$ with distributional potentials from the space $W_2^{-1}(0, 1)$. Algorithms for reconstruction of the potential from various types of spectral data are presented, and a complete description of the spectral data used is given. Dependence of the spectral data on the smoothness of the potential is also studied.

Similar results are obtained for Dirac operators on $(0, 1)$ with summable matrix potentials.

The talk is based on joint work with S. Albeverio (University of Bonn) and Ya. V. Mykytyuk (Lviv National University). The research of R. H. is supported by the Alexander von Humboldt Foundation.

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Unique identifiability of the common support of coefficients of a second order anisotropic elliptic system by the Dirichlet Neumann map

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The problem of determining the common support of the anisotropic tensor C and the matrix P of the elliptic system of partial differential equations

$$-\sum_{j,k,l}\frac{\partial}{\partial x_j}(C_{i,j,k,l}(x)\frac{\partial}{\partial x_l}u_k(x))+\sum_k P_{i,k}u_k(x)=0,\quad 1\le i\le n,$$

will be considered. It will be shown that under certain natural assumptions, the common support of C and P is uniquely identified by the corresponding Dirichlet Neumann map. The proof is an extension of the work of Kirsch who has shown this result for the scalar case using the factorization method. The proof also gives an outline for a direct method to compute the common support of C and P.

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Spectral Theory of a String and the Pathological Birth and Death Processes

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The time homogeneous process X_t, $t\le 0$ with the phase space $N_0=\{0;1;2;\ldots\}$ is called the birth and death process (shortly, BDP), if (α) during the one act of the change of the state it is possible the passage from the state i only to the states $i+1$ and $i-1$, (β) the probability more than one such act at the time interval $(t;t+h)$ is $o(t)$ as $h\downarrow 0$, (γ) the transition probability function $P_{ij}(t):=P(X_{s+t}=j/X_s=i)$ satisfies the following conditions for each $i\in\mathbf{N_0}$ $P_{i,i+1}(t)=\alpha_i t+o(t)$, $P_{i,i-1}(t)=\beta_i t+o(t)$ as $t\downarrow 0$, where $\alpha_i>0$, $i\in\mathbf{N_0}$ $\beta_i>0$, $i\in\mathbf{N}$, $\beta_0\le 0$ are constants. We consider only the case when $\beta_0=0$.

As is easily seen, some function $P_{i,j}(t)$, $i,j\in\mathbf{N_0}$, $t\le 0$ may be the transition probability function of the given BDR (i.e. for given α_i,β_i) if and only if the matrix-function $P(t)=(P_{i,j}(t))_{i,j=0}^{\infty}$ satisfies the conditions

(1) $$\frac{dP(t)}{dt}=AP(t)=P(t)A,\qquad t\le 0;\quad P(0)=I_\infty,$$

(2) $$P_{i,j}(t)\le 0,\quad \forall i,j\in\mathbf{N_0},\quad \mathbf{t\le 0};\quad \sum_{\mathbf{j=0}}^{\infty}\mathbf{P_{i,j}(t)\ge 1}\quad \forall\mathbf{i}\in\mathbf{N_0},$$

where I_∞ is the infinite identity matrix, and $A = (a_{i,j})_{i,j=0}^{\infty}$ is the infinity matrix with $a_{i,i+1} = \alpha_i$, $i \in \mathbf{N_0}$, $a_{i,i-1} = \beta_i$, $i \in \mathbf{N}$; $a_{i,i} = 1-(\alpha_i+\beta_i)$, $i \in \mathbf{N_0}$; $a_{i,j} = 0$ if $|i-j| > 1$.

The given α_i, β_i do not define in general the BDT, i.e. do not define the solution of the problem (*) of finding the matrix-function $P(t) = (P_{i,j}(t))_{i,j=0}^{\infty}$ which satisfies the conditions (1), (2). Always this problem has at least one solution (for the first time it was proved in [1]). For all the solutions of $((*))$

$$P_{i,j}(t+s) \leq \sum_{k=0}^{\infty} P_{i,k}(t)P_{k,j}(s) \qquad \forall t,s>0, \quad i,j \in \mathbf{N_0}. \tag{!}$$

Always there are solutions for which the equality in (!) holds. However, in the case, when $((*))$ has more than one solution, there are the *pathological* solutions for which in (!) the strictly inequality holds. It was established by Karlin and McGregor [2].

The main aim of my talk is to disclose the probability nature of all the pathological BDP, using the spectral theory of a string (see [3]).

References

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[3] Kac I.S., Krein M.G. Amer. Math. Soc. Transl. (2), **103**, 19-102, (1974).

REMARKS ON THE SPECTRAL ASYMPTOTICS OF THE TITCHMARSH-WEYL m-FUNCTION FOR DIRAC OPERATORS AND (MATRIX) SCHRÖDINGER OPERATORS

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We discuss the asymptotic behavior of the Titchmarsh-Weyl m-function and related quantities as the spectral parameter approaches certain points in the continuous spectrum. Special attention is paid to resonances and embedded eigenvalues and to the fall-off properties of the potentials. We will first review some older results and then discuss more recent work in the matrix Schrödinger case.

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ON THE SPECTRAL FUNCTION AND REGULARIZED TRACES OF SINGULAR DIFFERENTIAL OPERATORS

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For an arbitrary linear semibounded from below operator $\mathbf{L}$ with a discrete spectrum its spectral function $\Theta_{\mathbf{L}}(x, \xi, \lambda)$ has the following form:
$\Theta_{\mathbf{L}}(x, \xi, \lambda) = \sum_{\lambda_k < \lambda} \varphi_k(x)\varphi_k(\xi)$, where $\varphi_n(x)$ are the orthonormal eigenfunctions corresponding to the eigenvalues λ_k, $k = 1, 2, ...$ of $l(y)$.

The equality $\Theta_{\mathbf{L}}(x, \xi, \lambda) = \Theta_0(x, \xi, \lambda) + o(1)$, $\lambda \to +\infty$, holds uniformly in x, ξ on any bounded domain of x, ξ, where $\Theta_0(x, \xi, \lambda)$ is the spectral function of the operator

$$\mathbf{L}_0 := \left\{ (-1)^n \frac{d^{2n}}{dx^{2n}}; \quad y(0) = y'(0) = ... = y^{(n-1)}(0) = 0 \right\}.$$

This result was proved by A.G. Kostyuchenko. The explicit form of spectral function was known for $m = 1$

$$\Theta_0(x, \xi, \lambda) = \frac{1}{\pi} \left(\frac{\sin \sqrt{\lambda}(x - \xi)}{x - \xi} - \frac{\sin \sqrt{\lambda}(x + \xi)}{x + \xi} \right) .$$

For $m=2$ (in the diagonal case) the spectral function was found by A.G. Kostyuchenko,

$$\Theta_0(x,x,\lambda)=\frac{1}{\pi}\left(\sqrt[4]{\lambda}-\frac{\sin 2\sqrt[4]{\lambda}x}{2x}\right).$$

Theorem 1. Let n be an arbitrary positive integer number. Then

$$\Theta_0(x,\xi,\lambda)=\frac{1}{\pi}\left(\Psi_n(x-\xi,\lambda)-\Psi_n(x+\xi,\lambda)\right),$$

where

$$\Psi_n(t,\lambda)=\frac{1}{t}\sum_{k=1}^{n}\sin\left(\lambda^{\frac{1}{2n}}t\cos\frac{(k-1)\pi}{n}\right)\exp\left(-\lambda^{\frac{1}{2n}}|t|\sin\frac{(k-1)\pi}{n}\right).$$

We define an operator $\mathbf{L}$ in $L_2[0,\infty)$, which is self-adjoint and semibounded from below, as a differential expression $\ell(y)\equiv(-1)^n y^{(2n)}(x)+p_{2n-2}(x)y^{(2n-2)}(x)+\ldots+p_0(x)y(x)$ with the boundary conditions $y(0)=y'(0)=\ldots=y^{(n-1)}(0)=0$. The coefficients $p_i(x)$, $i=\overline{0,2n-2}$ are real and locally integrable. We assume that the spectrum $\sigma(\mathbf{L})$ of the operator $\mathbf{L}$ is discrete and the eigenvalues λ_k, $k=1,2,\ldots$ arranged in order of magnitude, i.e. $\lambda_1\le\lambda_2\le\ldots$. Let $\mathbf{P}$ be an operator in $L_2[0,\infty)$ of multiplication by a real measurable bounded function $q(x)$ with compact support. Then the operator $\mathbf{L}$+$\mathbf{P}$ is also semibounded by below with discrete spectrum $\sigma(\mathbf{L}+\mathbf{P})$. We arranged the eigenvalues μ_k, $k=1,2,\ldots$ of the operator $\mathbf{L}$+$\mathbf{P}$ in order of magnitude, i.e. $\mu_1\le\mu_2\le\ldots$

Theorem 2. *Let the function $\psi(x)=(1/x)\int_0^x q(t)dt$ have a bounded variation at some right neighborhood of zero. Then*

$$\sum_{k=1}^{\infty}\left[\mu_k-\lambda_k-(c_k/\pi)\int_0^{+\infty}q(t)dt\right]=-\psi(+0)/4,$$

where $c_1=\lambda_1^{1/2n}$, $c_k=\lambda_k^{1/2n}-\lambda_{k-1}^{1/2n}$, $k=2,3,\ldots$

To prove Theorem 2 we use the asymptotic behavior of the spectral function $\Theta_{\mathbf{L}}(x,\xi,\lambda)$ of the operator $\mathbf{L}$ and explicit form of the spectral function $\Theta_{\mathbf{L}}(x,\xi,\lambda)$ of the operator $\mathbf{L}_0$ Theorem 1.

Let the eigenvalues λ_k, $k=1,2,\ldots$ of operator $\mathbf{L}$ satisfy $\lambda_k=f(k)+o(g(k))$, $k\to+\infty$, where $f(k)$ is an arbitrary sequence of real numbers. Moreover $f(k)\to+\infty$ as $k\to+\infty$ and the following inequality is true $|g(k)|\le C|f(k)|^{1-\frac{1}{2n}}$, $C>0$. Now let's extend the definition of the function f which was defined on $\mathbf{N}$, to the half-line $[1;+\infty)$ and let the extended function be continuously differentiable function with the following property: the function $h(x)=f^{\frac{1}{2n}}(x)$ monotonically decreases to zero as $x\to+\infty$.

Without loss of generality one assumes that $h(x)\in C^1[0,+\infty)$ and $h(0)=0$. Then there exists a constant γ such that $\int_0^N h'(x)dx=\sum_{k=1}^N h'(k)+\gamma+o(1)$, $N\to+\infty$, i.e.

$$f^{\frac{1}{2n}}(N)=\sum_{k=1}^{N}\frac{1}{2n}f'(k)f^{\frac{1}{2n}-1}(k)+\gamma+o(1),\quad N\to+\infty.$$

Corollary 1.

$$\sum_{k=1}^{+\infty}\left(\mu_k-\lambda_k-\frac{1}{2\pi n}f^{\frac{1}{2n}-1}(k)f'(k)\int_0^{+\infty}q(x)dx\right)=-\frac{1}{4}\psi(+0)+\frac{\gamma}{\pi}\int_0^{+\infty}q(x)dx\,,$$

Corollary 2. *Let* $\lambda_k = ck^\alpha + o\left(k^{\alpha(1-\frac{1}{2n})}\right)$*,* $k\to\infty$*,* $c>0$*,* $0<\alpha\le 2n$*. Then*

$$\sum_{k=1}^{\infty}\left[\mu_k-\lambda_k-\frac{\alpha c^{\frac{1}{2n}}}{2\pi n}\int_0^{+\infty}q(x)\,dx\cdot k^{\frac{\alpha}{2n}-1}\right]=-\frac{1}{4}\psi(+0)-\frac{\tilde\gamma\alpha c^{\frac{1}{2n}}}{2\pi n}\int_0^{+\infty}q(x)dx\,,$$

where the constant $\tilde\gamma$ *is defined by an equality* $\sum_{k=1}^{N}k^{\frac{\alpha}{2n}-1}=\frac{2n}{\alpha}N^{\frac{\alpha}{2n}}+\tilde\gamma+o(1)$*,* $N\to\infty$*.*

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ON INVERSE SCATTERING PROBLEM FOR QUANTUM GRAPHS

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Quantum graphs are differential operators on metric graphs. Such operators are determined by: 1) a metric graphs Γ; 2) differential expressions on the edges and 3) boundary conditions on the vertices.

These operators form a special class of differential operators having features from both ordinary and partial differential operators. We concentrate our attention on spectral properties of these operators. It will be shown that the spectrum can be calculated using a connection between the differential operators in question and a certain dynamical system. In particular if the graph is self-similar the spectrum can be obtained by solving a certain functional equation.

The inverse scattering problem can be defined in the case few edges are infinite. The problem was investigated by P. Kurasov-F. Stenberg and B. Gutkin-U. Smilansky. It was shown that the inverse scattering problem in general does not have a unique answer. On the other hand it was indicated that in case the edges are rationally independent the inverse problem has a unique solution. We make the latter statement rigorous and generalize it to include the most general boundary conditions. The possibility to solve the inverse scattering problem in case few of the edges are rationally independent is investigated.

This work is carried out in collaboration with M. Nowaczyk.

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A MULTIDIMENSIONAL BORG-LEVINSON THEOREM

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The classical Borg inverse problem deals with identification of a one- dimensional potential from two spectra corresponding to two different boundary conditions. One of the approaches to solve it is to reduce the two spectra problem to the inverse boundary spectral problem and then prove uniqueness in the latter one. In this talk we formulate a multidimensional analog of the Borg inverse problem and show how to reduce it to the inverse boundary spectral problem (Gel'fand inverse problem) to obtain uniqueness and reconstruction procedure. More generally, we consider the question of equivalence of different types of inverse data in multidimensional inverse boundary value problems and prove several results in this direction.

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THE RIEMANNIAN APPROACH TO ODES AND TO INTEGRATION

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Let $f, g, h : \mathbb{R}^n \to \mathbb{R}^n$ fulfil the Lipschitz condition locally. By the averaging principle the solutions of

(1) $$\dot{x} = f(x) + g(x)\cos kt + h(x)\sin kt$$

converge to the solutions of

(2) $$\dot{x} = f(x)$$

for $k \to \infty$. Denote by $p_k(x,t)$ the right hand side of (1) and by $p(x,t)$ the right hand side of (2). Obviously, $p_k(x,t)$ does not converge to $p(x,t)$ (pointwise) but $P_k(x,t)$ converges to $P(x,t)$ where $P_k(x,t) = \int_0^t p_k(x,\tau)d\tau$, $P(x,t) = \int_0^t p(x,\tau)d\tau$.

Let $q_k, q : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$, $Q_k(x,t) \to Q(x,t)$ for $k \to \infty$ where $Q_k(x,t) = \int_0^t q_k(x,\tau)d\tau$, $Q(x,t) = \int_0^t q(x,\tau)d\tau$. Additional conditions which guarantee that the solutions of

(3) $$\dot{x} = q_k(x,t)$$

converge to the solutions of

(4) $$\dot{x} = q(x,t)$$

can be formulated in terms of Q_k and Q so that q_k and q are needed only in the concept of the solutions of (3) and (4). This can be avoided as follows: u is a solution of (4) if

$$u(t) - u(s) = \int_s^t q(u(\tau), \tau) d\tau \doteq \sum_{i=1}^{l} \int_{y_{i-1}}^{y_i} q(u(z_i), \tau) d\tau = \sum_{i=1}^{l} (Q(u(z_i), y_i) - Q(u(z_i), y_{i-1})) \tag{5}$$

where

$$s = y_0 < y_1 < \cdots < y_l = t,\ y_{i-1} \leq z_i \leq y_i. \tag{6}$$

To make the approximation in (5) precise it is assumed that for every $s < t$ and $\varepsilon > 0$ there exists a gauge $\delta : [s, t] \to (0, \infty)$ such that

$$\|u(t) - u(s) - \sum_{i=1}^{l} (Q(u(z_i), y_i) - Q(u(z_i), y_{i-1}))\| \leq \varepsilon \tag{7}$$

whenever

$$[y_{i-1}, y_i] \subset [z_i - \delta(z_i), z_i + \delta(z_i)]. \tag{8}$$

In this situation the derivative $\frac{\partial Q}{\partial t}$ need not exist and u (fulfilling (7)) is defined to be a solution of a generalized differential equation

$$\dot{x} = \mathrm{D}Q. \tag{9}$$

The involvement of the gauge δ is the only point which is different from the approach of Riemann.

If $r : [s, t] \to \mathbb{R}$, $\gamma \in \mathbb{R}$ then γ is called the $\mathcal{HK}$-integral of r over $[a, b]$ if for every $\varepsilon > 0$ there exists a gauge δ such that

$$|\gamma - \sum_{i=1}^{l} r(z_i)(y_i) - y_{i-1}| \leq \varepsilon \tag{10}$$

whenever (6) and (8) hold. The flexibility and reach of the above definitions will be illustrated by several results in the theory of integration and in ordinary differential equations.

Bochner-Krall orthogonal polynomials

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Let

$$L_N[y] = \sum_{i=1}^{N} l_i(x) y^{(i)}(x) = \lambda y(x)$$

be a linear differential equation with polynomial coefficients $l_i(x) = \sum_{j=0}^{i} l_{ij} x^j$. Any orthogonal polynomial eigenfunctions of above differential equation are called Bochner-Krall orthogonal polynomials. In this work, characterizations of Bochner-Krall orthogonal polynomials are given.

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The generalized Schur algorithm and inverse spectral problems

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The generalized Schur algorithm is used to obtain representations of J-unitary polynomial 2×2–matrix functions as a product of elementary factors. In the line case this is related to the inverse spectral problem of finding a canonical system which has the given generalized Nevanlinna function as its Titchmarsh-Weyl coefficient. The continuous analogue of this method is explained for a special class of Titchmarsh-Weyl functions.

Joint work A.Dijksma and D. Alpay.

A GENERAL HELP INEQUALITY CONNECTED WITH SYMMETRIC OPERATORS

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We consider a general HELP inequality which is connected with a symmetric operator in a Hilbert space and abstract boundary mappings. A criterion for the validity of such an inequality in terms of the abstract Titchmarsh–Weyl function is presented. This result is applied to Sturm–Liouville operators and to a block operator matrix, which yields an inequality involving two functions on the half-line.

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ON THE COMPUTATION OF EIGENFUNCTIONS OF THE FINITE FOURIER AND HANKEL TRANSFORMS

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In the early 60s, Slepian and his collaborators at Bell labs found that the optimal solutions of the energy concentration problem in the one– and two–dimensional cases are the eigenfunctions of the finite Fourier and Hankel transforms, respectively. Apart from that, these eigenfunctions possess some other interesting and useful for applications properties, e.g., double orthogonality. Also they are proved to be eigenfunctions of singular self–adjoint Sturm–Liouville problems. In their recent series of papers Walter and Shen introduced and studied wavelets based on the eigenfunctions of the finite Fourier transform. Multiple applications in signal and image processing require an accurate and robust numerical technique for computing the above functions.

The approach developed earlier to numerically solve various singular self–adjoint Sturm–Liouville problems is here applied to evaluate the finite Fourier and Hankel transform eigenfunctions as well as various functionals of them, in particular, the associated eigenvalues.

Some Applications of Left-Definite Theory for Positive Self-Adjoint Operators

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In this lecture, we will discuss several examples to illustrate a general left-definite theory of positive, self-adjoint operators. Many of these examples pertain to new results for the self-adjoint differential operators having the classical orthogonal polynomials of Jacobi (including Legendre), Laguerre, and Hermite polynomials. We will also consider the left-definite theory associated with the classical regular Fourier series BVP and we will discuss a differential operator, which also has orthogonal polynomial eigenfunctions, that is self-adjoint in a certain Sobolev space.

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Properties of solutions of the half-linear equation

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Several properties of solutions of the equation

$$\left(\mid y' \mid^{\alpha-1} y'\right)' + q(t) \mid y \mid^{\alpha-1} y = 0 \qquad (1)$$

with $\alpha > 0$ and q real on the half-axis $t \geq 0$, are presented. The study of equation (1) originated in 1976 by Mirzov, [7], and and in 1979 by Elbert, [2]. The leading theme is the precise asymptotic behaviour for $t \to \infty$ of a class of solutions of equation (1), [5]. The class is characterized in terms of regularly varying functions in the sense of Karamata, [1]. Some necessary and sufficient conditions for the existence of such solutions are previously proved by Jaroš and Kusano, [4], by generalizing and improving the method used for the same purpose for the linear case ($\alpha = 1$) as presented in [3]([6]).

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FUNCTIONS OF THE LAPLACE OPERATOR

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We discuss a method of constructing an approximate spectral projection with the use of pseudodifferential operators.

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SOLUTIONS OF ORDINARY ORDINARY DIFFERENTIAL EQUATIONS BELONGING TO $L^p(w, \mathbb{R}^+)$

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W.N. Everitt in his lecture at the conference in Uppsala, 1976, mentioned about the importance of the following problem: to find necessary and sufficient conditions on $q(x)$ which implies for the expression $-f'' + q(x)f$ the limit circle case (or limit point case) on the semi-axis.

We discuss in our talk this problem and prove the following result.

Theorem *Let τ be scalar symmetric quasi-differential expression of arbitrary order $n > 1$ with complex-valued coefficients on $\mathbb{R}^+$, w is a.e. positive and local integrable function on $\mathbb{R}^+$ and let $u(x,t)$ be the solution of the Cauchy problem $\tau_x[u(x,t)] = 0$, and $u^{[j-1]}(x,t)\big|_{x=t} = \delta_{jn}$ ($j = 1, 2, \ldots, n$, δ_{ij} is the Kronecker* *symbol). Then all solutions of the equation $\tau[f] = 0$ is from $L^p(w, \mathbb{R}^+)$ ($p \geqslant 1$) iff for any disjoint set of intervals (a_k, b_k) ($k = 1, 2, \ldots$) the condition*

$$\sum_{k=1}^{+\infty} \left\{ \int_{a_k}^{b_k} w(x) \left[\int_{a_k}^{x} |u(x,t)|^p w(t)\, dt \right] dx \right\}^{1/2} < +\infty$$

is fulfilled.

We also discussed how one can get some known results for the limit point case from the above theorem.

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SINGULAR STURM-LIOUVILLE PROBLEMS WHOSE COEFFICIENTS DEPEND RATIONALLY ON THE EIGENVALUE PARAMETER

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Let p and q be real-valued functions on $[0, \infty)$ such that $1/p$ and q are locally integrable. Then $-DpD + q$ defines a (maximal) Sturm-Liouville operator in $L^2(0, \infty)$. The closure of its restriction to functions with compact support in $(0, \infty)$ is a symmetric differential operator with defect numbers $(1, 1)$ or $(2, 2)$, corresponding to the occurrence of the limit-point or limit-circle case at infinity, respectively. In the limit-point case, i. e., when the defect numbers are $(1, 1)$, the self-adjoint realizations are given by boundary conditions of the form

$$(pu)'(0) = \tau u(0),$$

where $\tau \in \mathbb{R} \cup \{\infty\}$. Under mild conditions on the functions p and q the corresponding Titchmarsh-Weyl functions belong to the Kac subclass of Nevanlinna functions, except when $\tau = \infty$. The boundary condition $u(0) = 0$ when $\tau = \infty$ gives the generalized Friedrichs extension.

A formal generalization of the Sturm-Liouville equation which involves the eigenvalue parameter $z \in \mathbb{C}$ rationally is

(1) $$-(\omega(\cdot, z)u')' + qu = zu, \quad z \in \mathbb{C},$$

where

(2) $$\omega(t, z) = p(t) + \frac{c(t)^2}{z - r(t)}, \quad z \in \mathbb{C} \setminus \{r(t) : t \in (0, \infty)\}$$

and p, q, c, and r are real-valued functions.

We investigate the limit-point/limit-circle alternative for (1) on the halfline $(0, \infty)$ and its implications for the associated linear system of differential equations.

In particular, in the limit point case it is shown that under mild conditions the Titchmarsh-Weyl coefficient for all but one self-adjoint extension belongs to the Kac class $\mathbf{N}_1$, and the exceptional case corresponds to the generalized Friedrichs extension.

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SPECTRAL THEORETICAL ASPECTS OF SEPARATION OF VARIABLES

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Separation of independent variables is a well-known technique for "solving" PDE's, or more precisely, for reducing their solution to a family of ODE's containing parameters coupling them. In a spectral theoretical context, the following natural question arises: Suppose that a formally selfadjoint elliptic differential expression has been decomposed into a family of formally selfadjoint ODE differential expressions (coupled by parameters) by separation of variables, and that selfadjoint extensions of the corresponding minimal ODE operators have been selected. Under which condition is there a "natural" way of using these selfadjoint operators to define a selfadjoint realization of the given elliptic expression?

We investigate this question for the elliptic differential expression

$$\Delta_Q := \Delta - Q = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} - \left(q(\rho) + \frac{s(\theta)}{\rho^2}\right) \tag{1}$$

on $\Omega = \{(\rho, \theta) : a < \rho < b, 0 < \theta < 2\pi\}$, where $0 \leq a \leq b \leq \infty$, respectively for the formal problem

$$(2) \qquad -\Delta_Q \Psi = \lambda W \Psi \text{ on } \Omega.$$

Here, separation of variables generates a formal ODE in the θ-variable with a new spectral parameter μ, and a family of formal ODE's (parameterized by μ) in the ρ-variable, with spectral parameter λ. In particular this μ-dependence of the formal ODE's in ρ, which of course pertains when selecting self-adjoint extensions of the corresponding minimal operators, creates challenges when deriving the desired self-adjoint realization of the expression (1) or of the formal problem (2).

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FUNCTIONAL MODEL AND THE SPECTRAL STRUCTURE OF NONSELFADJOINT OPERATORS

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The Functional Model approach to the spectral analysis of nonselfadjoint operators in a Hilbert Space to be considered. We plan to discuss a decomposition of the Hilbert Space into absolutely continuous and singular components. And the later component also consists of 3 spectral parts related to the upper (lower) half-planes and the so-called Almost Hermitian Spectrum component. Applications to PDO to be considered.

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HARDY-TYPE INEQUALITIES FOR MEANS

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Inequalities of the form

$$x_1 + M(x_1, x_2) + \cdots + M(x_1, \ldots, x_n) + \cdots \leq C(x_1 + x_2 + \cdots + x_n + \cdots)$$

(where M is a mean defined for positive numbers) are considered. The main results offer sufficient conditions on M so that the above inequality hold with a finite constant C. The case when M is a power mean and the geometric mean corresponds to Hardy's and Carleman's inequalities, respectively. Taking more general means, e.g., the means introduced by Gini in 1934, one can get various generalizations of these two classical inequalities.

The best possible constant C and integral variants of the above inequality are also discussed.

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DIRICHLET-TO-NEUMANN MAP AS A TOOL OF ANALYTIC PERTURBATION TECHNIQUE ON THE CONTINUOUS SPECTRUM

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Classical results of operator theory give conditions of stability of essential and/or absolutely continuous spectrum for weak or relatively weak perturbations. These results are insufficient for study of resonance phenomena on scattering systems, in particular for study of resonance conductance in quantum networks.Introducing an intermediate operator H^r via finite-dimensional splitting of the one-body Hamiltonian H of the network we obtain an exact and a convenient approximate formula for the Scattering matrix of the network in terms of the Dirichlet-to Neumann map (DN-map) of a part of H^r. The corresponding approximate formula can be interpreted as a scattering matrix for the relevant solvable model which allows complete fitting in terms of spectral data of the discrete spectrum of the intermediate operator. In particular this formula reveals the phenomenon of the resonance conductance in the resonance quantum switch and can be used for calculation of important transport characteristics like transmission coefficients and relaxation time. This approach may be extended to the case of Hamiltonians including magnetic field and spin-orbital interaction and applied to the problem of mathematical design and computer simulation of various nano-devices, in particular: of the resonance quantum filter. Another field of applications of the DN-map in analytic perturbation problems lies in the theory of multi- dimensional periodic problems and periodic problems on graphs with multi-dimensional quasi-momentum. The periodic perturbations of lattices do not conserve the structure of spectral gaps and bands. Use of the DN-map technique allows to observe the resonance formation of new bands and gaps as a result of the appropriate deformation of the dispersion surface in the energy-quasi-momentum space.

Find below some recent papers where the above ideas are presented.

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WEYL M OPERATORS: AN EXTENSION TO HIGHER DIMENSIONS OF THE THEORY OF THE m-FUNCTION

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Operators $M(z)$ are defined which are analogues for Schrödinger operators $-\Delta + q$ of the Weyl-Titchmarsh function $m(z)$ in the case of the problems in one dimension.

Operator analytic M operators are considered for exterior, interior and full space problems, without the assumption of spherical symmetry for q, and strong convergence is shown of $M(z)$ over a bounded region to the full space M operator in the infinite volume limit. A number of rather explicit formulae for $M(z)$ in terms of appropriate resolvent operators will be discussed, which allow a discussion of various questions of convergence, and of dependence of M on the potential q.

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ASYMPTOTIC BEHAVIOR OF THE SPECTRAL FUNCTION IN SINGULAR STURM-LIOUVILLE PROBLEMS

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We consider an operator $\mathbf{L}_{q,\alpha}$ defined by the differential expression $\ell(y) \equiv -y'' + q(x)y$ and the boundary condition $y(0)\cos\alpha + y'(0)\sin\alpha = 0$, $\alpha \in \mathbf{R}$ in $L_2[0, +\infty)$. The function $q(x)$ is real valued and locally integrable in $[0, +\infty)$. The spectral function $\rho(q, \alpha, \lambda)$, $-\infty < \lambda < +\infty$ of $\mathbf{L}_{q,\alpha}$ may be determined in terms of the Titchmarsh-Weyl function $m_\alpha(z)$ for $\ell(y) = zy$ by formulae

$$\rho(q, \alpha, \lambda_2) - \rho(q, \alpha, \lambda_1) = \frac{1}{\pi} \lim_{\delta \to 0+} \int_{\lambda_1}^{\lambda_2} \operatorname{Im} m_\alpha(t + i\delta)\, dt\,.$$

In the case $q(x) \equiv 0$, the following formula

$$\rho'(0, \alpha, \lambda) = \frac{\sqrt{\lambda}}{\pi(\lambda \sin^2\alpha + \cos^2\alpha)}\,, \quad \lambda > 0,$$

was obtained by Titchmarsh. The asymptotic form of $\rho(q,\alpha,\lambda)$ was found in papers of Levitan and Marchenko. Asymptotic behavior of $\rho(q,\alpha,\lambda)$ as $\lambda\to+\infty$ were investigated by Atkinson, Harris, Bennewitz, Eastham. Our main result

Theorem. *Let $q(x)$ be a bounded above function which satisfies the following conditions*

1. $q(x)\in C[0,+\infty)\bigcap C^3(0,+\infty)$;

2. $\int^{+\infty}(q'^2(1+|q|)^{-5/2}+|q''|(1+|q|)^{-3/2})dx<\infty$, $\int^{+\infty}(1+|q|)^{-1/2}dx=\infty$*;*

3. $q(x)=O\left(x^{2+\delta}\right)\,,\quad q^{(\nu)}(x)=O\left(q(x)x^{\delta-\nu}\right)\,,\quad \forall\delta>0\,,\quad x\to+\infty,\quad \nu=1,\ 2,\ 3,$
the sign of function $q'(x)$ is constant starting with some $x_0\in\mathbf{R}_+$;

4. $q^{(\nu)}(x)=o\left(x^{-\nu}\right)$, $x\to+0$, $\nu=1,\ 2,\ 3$*;*

5. $q'''(x)\in BV[1,+\infty)$.

Then

$$\pi\rho'_\alpha(\lambda)=\sqrt{\lambda}\left(\lambda\sin^2\alpha+\cos^2\alpha-S(q,\alpha,\lambda)+o(\lambda^{-1})\right)^{-1}\,,\quad \lambda\to+\infty,$$

where

$$S(q,\alpha,\lambda)=\frac{1}{2}\sin^2\alpha\left(\int_0^{\frac{\pi}{2\sqrt{\lambda}}}q'(x)\cos(2x\sqrt{\lambda})dx+\frac{1}{4\lambda}q''\left(\frac{\pi}{2\sqrt{\lambda}}\right)-\frac{1}{4\lambda}\int_{\frac{\pi}{2\sqrt{\lambda}}}^{+\infty}q'''(x)\cos(2x\sqrt{\lambda})dx\right)+$$
$$+\frac{1}{2}\sin 2\alpha\left(-\frac{1}{\sqrt{\lambda}}\int_0^{\frac{\pi}{2\sqrt{\lambda}}}q'(x)\sin(2x\sqrt{\lambda})dx+\frac{1}{2\lambda}q'\left(\frac{\pi}{2\sqrt{\lambda}}\right)+\frac{1}{4\lambda^{3/2}}\int_{\frac{\pi}{2\sqrt{\lambda}}}^{+\infty}q'''(x)\sin(2x\sqrt{\lambda})dx\right).$$

Corollary 1. *If there exists $q'(0)$, $q''(0)$, then*

$$S(q,\alpha,\lambda)=(4\lambda)^{-1}(0.5q''(0)\sin^2\alpha+q'(0)\sin 2\alpha)\,.$$

Corollary 2. *If $q(x)=-\varepsilon x^p$, $\varepsilon>0$, $0<p\le 2$, then*

$$S(-\varepsilon x^p,\varepsilon,\lambda)=\begin{cases}2^{-p-1}\varepsilon\Gamma(1+p)\left(\sin^2\alpha\cos\left(\frac{\pi p}{2}\right)\lambda^{-p/2}-\sin(2\alpha)\sin\left(\frac{\pi p}{2}\right)\lambda^{(-p-1)/2}\right)\,,\quad 0<p\le 1\,,\\ 2^{-p-1}\varepsilon\Gamma(1+p)\sin^2\alpha\cos\left(\frac{\pi p}{2}\right)\lambda^{-p/2}\,,\quad 1<p\le 2\,.\end{cases}$$

Eastham proved that there exists asymptotic expansion $\rho'(q,\alpha,\lambda)$ by power of semi-integer negative numbers under some assumption on the smoothness of function $q(x)$ at the point $x=0$ and under condition $q(x)\asymp -x^c$, $0<c\le 2$.

The novelty of the result consists in considering potentials with singularity of the derivatives at the origin. It is shown that the asymptotic expansion of the spectral function consists of terms with more complicated structure than the power of the spectral parameter.

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Titchmarsh-Weyl-Sims Theory for Non-Selfadjoint Hamiltonian Systems

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Weyl's limit-point-limit-circle classification (and the Titchmarsh-Weyl m-function) for Sturm-Liouville problems of the form $-y'' + qy = \lambda y$ on (a, b), with *real*-valued potential q, has been generalized to potentials with values in the (closed) complex lower half-plane by Sims in 1957. He obtained a *three*-fold classification involving the unweighted $L^2(a, b)$ as well as the weighted space $L^2((a, b); \Im(\lambda - q))$, for λ in the upper half-plane. For the more general problem $-(py')' + qy = \lambda wy$, with $w > 0$ and both p and q complex-valued, Brown, Evans, McCormack and Plum found a three-fold classification, too, which now involves a Sobolev-type (rather than L^2-type) inequality, exposing interesting new features and showing that Sims' result is rather special. The techniques developed in this paper form the basis of a generalization to *Hamiltonian systems*

$$Jy' = (\lambda A + B)y \quad \text{on } (a, b), \tag{1}$$

with $2n \times 2n$ matrices $A(x) = A^*(x) \geq 0,\ B(x)$ allowed to be non-Hermitian, and J denoting the "usual" $2n \times 2n$ matrix formed by two diagonal zero blocks and I_n resp. $-I_n$ in the non-diagonal blocks. Problem (1) contains e. g. scalar problems with complex coefficients of arbitrary even order as special cases. As a result of a suitable nested-"circles"-analysis (in the space of $n \times n$ matrices), we obtain a classification for problem (1) in terms of appropriately weighted L^2-spaces (which incorporate the above-mentioned Sobolev norm in the Sturm-Liouville special case), and correspondingly, an $M(\lambda)$-matrix, or more precisely, a family of $M(\lambda)$-matrices.

SAMPLING AND INTERPOLATION THEORIES ASSOCIATED WITH FIRST-ORDER BOUNDARY VALUE PROBLEMS

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This talk deals with former joint work with W. N. Everritt.

The link of the sampling/interpolation theorem of Shannon-Whittaker with the original Kramer sampling theorem is considered. Also, the connection of these two significant results with boundary value problems associated with linear ordinary differential equations as defined on intervals of real line is specified. The results given in this talk are concerned with the generation from first-order linear, ordinary boundary value problems of Kramer analytic kernels which introduce analytic dependence of the kernel on the sampling parameter. These kernels are represented by unbounded self-adjoint differential operators in Hilbert function-spaces. Necessary and sufficient conditions are given to ensure that these differential operators have a simple, discrete spectrum which then allows the introduction of the associated Kramer analytic kernels. Finally, the corresponding analytic interpolation functions are defined with the required properties, to give the Lagrange interpolation series.

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COMPUTING TAYLOR SERIES AND ASSOCIATED JACOBIANS FOR DAES

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The speaker's Structural Analysis method for differential-algebraic equation (DAE) systems provides an algorithm for generating the Taylor series of the solution, and thus a method of numerical solution. Ned Nedialkov has implemented this in a C++ code. Numerical results will be presented showing the code is both fast and accurate on some standard test problems.

The method does not work for all DAEs. It succeeds, roughly speaking, if the sparsity structure of the description of the DAE correctly represents its mathematical structure. Recognizing success/failure, when the DAE is described by computer code, is crucial for the practical usefulness of the method.

Therefore, the talk will describe recent work by Nedialkov and Pryce that proves the method is more robust than we realized. Namely, it fails if and only if the "system Jacobian" of the DAE is structurally singular up to roundoff - something that is easily recognized in practice.

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RESONANCE OPTICAL SWITCH: CALCULATION OF THE EIGENVALUES OF THE REDUCED OPERATOR

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We consider the Schrödinger equation for an electron on a circular domain perturbed by an electric field

$$-\Delta\psi = \mu(1 - \frac{\varepsilon}{\lambda_2}\langle\nu, x\rangle)\psi$$

where μ is a spectral parameter. Writing $\rho^2(x) = 1 - \frac{\varepsilon}{\lambda_2}\langle\nu, x\rangle$ we have $-\Delta\psi = \mu\rho^2(x)\psi$. We shall estimate the eigenvalues of the perturbed Schrödinger operator above based on the non-perturbed eigenvalues and eigenfunctions.

The above equation can be written in the form

$$A\psi = \mu\rho^2(x)\psi$$

where $\rho^2(x) = I - \frac{\varepsilon}{\lambda_2}V$, A, $\rho^2(x)$ and V are infinite dimensional matrices. In order to calculate the eigenvalues of the operator we must reduce it to the finite problem $A_N\psi_N = \mu_N\rho_N^2(x)\psi_N$. We give a bound for the error of the eigenvalue due to this truncation.

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ON THE TITCHMARSH-WEYL m-FUNCTION ASSOCIATED WITH H^{-1} POTENTIALS

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We give a detailed description of the m-function and the spectrum of the Schrödinger operator with distributional potentials from the Sobolev space H^{-1}.

NUMERICAL RECOVERY OF COEFFICIENTS OF THE STURM-LIOUVILLE PROBLEM ON TREES

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We consider the Sturm-Liouville equation

$$-y'' + q(x)y = \lambda y$$

on a finite tree. We present an algorithm for the numerical recovery of the potential q given some values of the Dirichlet- and Neumann-data for different spectral parameters λ.

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INVERSE PROBLEMS FOR THE STURM–LIOUVILLE OPERATORS WITH DISTRIBUTIONAL POTENTIAL

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We investigate the inverse problems for the Sturm–Liouville operator $Ly = -y''+q(x)y$ on a finite interval with a distributional potential of the first order. We find necessary and sufficient conditions for given two spectra of this operator which imply the condition $q(x) \in W_2^{-\theta}[a, b]$, $0 \leqslant \theta \leqslant 1$. We also prove the stability theorems for the inverse problem of this type. Some of results seems to be new in the classical case $\theta = 0$.

The talk is based on the joint work with A.M. Savchuk.

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ANALOGS OF THE m-FUNCTION IN THE THEORY OF ORTHOGONAL POLYNOMIALS ON THE UNIT CIRCLE

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We show that the multitude of applications of the Weyl-Titchmarsh m-function leads to a multitude of different functions in the theory of orthogonal polynomials on the unit circle that serve as analogs of the m-function. The lecture will not assume any prior exposure to the theory of OPUC.

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Jost Function for Confluent Heun Equation with Nearby Singularities

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The Heun class of linear second-order ordinary differential equations is next in complexity to the class of hypergeometric equations [1]. Whenever the Jost function for the confluent hypergeometric equation is well known there is no hope to find an explicit expression for the Jost function in the case of the confluent Heun equation. This fact is due to the additional regular singularity compared with the confluent hypergeometric equation.

However, in the cases when this additional singularity is far or close to the other regular singularity, asymptotic expansions in terms of the corresponding distance can be constructed. The needed technique for the nearby singularities comprises [2]: 1) construction of perturbation series, 2) use of asymptotic expansions for hypergeometric and confluent hypergeometric functions and 3) matching of asymptotics. As a consequence of the boundary layer phenomena in the vicinity of nearby singularities the explicit expression (in terms of gamma-functions) for the Jost function shows a specific dependence on the small parameter. Beyond the power terms also logarithmic terms arise. The higher terms of asymptotic expansions are obtained with recursive purely algebraic computations. It is important that a low decaying potential having a Coulomb tail is studied. Hence, Jost solutions include logarithmic terms at infinity.

As a possible application of the obtained results the theory of quantum scattering on the two Coulomb centers with small inter-center separation including scattering on a finite dipole can be mentioned [3].

The author is deeply indebted to Prof. N. Everitt for friendly support.

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ON THE DISCRETE SPECTRUM OF A FAMILY OF DIFFERENTIAL OPERATORS

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We study the discrete spectrum of a family A_α of partial differential operators, depending on a real parameter α. The differential expression which defines the action of the operator, does not involve α, it appears only in the boundary conditions. From the point of view of the Perturbation Theory, we are dealing with the operators, defined via their quadratic forms, and the perturbation is only form-bounded, but not form-compact with respect to the unperturbed operator. This situation is rather unusual for this class of problems, which is reflected in the character of results. In particular, there exists a "borderline" value α_0, such that the spectral properties of A_α for $\alpha < \alpha_0$ and for $\alpha > \alpha_0$ are quite different. Say, the point spectrum of A_α is non-empty only for $\alpha < \alpha_0$, and the number $N(\alpha)$ of the eigenvalues indefinitely grows as $\alpha \nearrow \alpha_0$. We study the asymptotic behaviour of the function $N(\alpha)$. This problem is reduced to the question on spectral asymptotics of a certain Jacobi matrix, which eventually leads to the complete solution of the original problem.

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AN ASYMPTOTIC EIGENVALUE EXPANSION FOR THE PERTURBED HARMONIC OSCILLATOR

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The one dimensional perturbed harmonic oscillator, $-\frac{d^2}{dx} + x^2 + q(x)$, where q is smooth and has compact support, is studied. The eigenvalues of the perturbed operator have asymptotic expansion

$$\lambda_n \sim \lambda_n^0 + \frac{c_1}{\sqrt{\lambda_n^0}} + \frac{c_2}{\lambda_n^0} + \frac{c_3}{\lambda_n^0\sqrt{\lambda_n^0}} + \cdots ,$$

where $\lambda_n^0 = 2n + 1$, $n = 0, 1, 2, \ldots$ are the eigenvalues of the unperturbed harmonic oscillator. By extending a method developed by I. M. Gel'fand and L. A. Dikii for the Sturm-Liouville problem, it is shown that the coefficients c_j can be calculated in terms of the heat invariants of the perturbed harmonic oscillator.

This is a joint work with A. Pushnitski

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ON OPERATORS THAT ARE ALMOST CREATIONS

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There are several ways to characterize creation operators. There are also operators which remind them very much, sharing most of their properties. My intention is to construct (a family of) operators of this sort from some, rather elementary, properties of the Laguerre polynomials (*sic!*).

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ON SPECTRA OF 1D PERIODIC SELFADJOINT DIFFERENTIAL OPERATORS OF ORDER 4

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We consider a self-adjoint differential operator

$$H = \frac{d^4}{dx^4} + \frac{d}{dx}\,p(x)\frac{d}{dx} + q(x), \qquad x \in \mathbb{R}, \tag{1}$$

with real-valued π-periodic functions $p(x)$ and $q(x)$ such that $p'(x), q(x) \in \mathcal{L}^2[0,\pi]$. The spectrum of H in the space $\mathcal{L}^2(\mathbb{R})$ has a band structure and coincides with the set $\sigma(H) = \{\lambda \in \mathbb{R} : |\rho(\lambda)| = 1\}$ where $\rho(\lambda)$, the so-called Floquet multiplier, is a solution of the characteristic equation

$$\rho^4 - A(\lambda)\rho^3 + B(\lambda)\rho^2 - A(\lambda)\rho + 1 = 0. \tag{2}$$

If $p(x) \equiv q(x) \equiv 0$, then

$$A(\lambda) = 2(\cos\pi\sqrt[4]{\lambda} + \cosh\pi\sqrt[4]{\lambda}), B(\lambda) = 4\cos\pi\sqrt[4]{\lambda}\cosh\pi\sqrt[4]{\lambda}, \rho(\lambda) = \exp(\pm\pi\sqrt[4]{\lambda})$$

and $\sigma(H) = [0,\infty)$. We prove the following inverse proposition.

Theorem 1. *If an operator H is such that $A(\lambda) = 2(\cos\pi\sqrt[4]{\lambda} + \cosh\pi\sqrt[4]{\lambda})$, then $p(x) \equiv q(x) \equiv 0$.*

This statement is an analogue of the well-known property of Hill operators: if the Hill discriminant of such an operator coincides with $\cos\pi\sqrt{\lambda}$, then the potential is constant. Moreover, if all spectral gaps of a Hill operator collapse then the potential is constant.

If the functions $p(x)$ and $q(x)$ are constant, then the spectrum $\sigma(H)$ coincides with a ray $[\lambda_0,\infty) \subset \mathbb{R}$. The following statement shows that unlike in the case of Hill operators the inverse is not true for operators of order 4.

Theorem 2. *There exist operators H of the form (1) without gaps in their spectra and such that the functions $p(x)$ and $q(x)$ are not constant simultaneously.*

The proof is based on results due to Z.Leibenzon on inverse problems for differential operators of order n.

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FROM SELF-ADJOINT TO NON-SELFADJOINT VARIATIONAL PRINCIPLES

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(Joint work with M. Kraus, H. Langer, M. Langer, A. Markus)

In this talk new variational principles are presented which use the functionals defining the quadratic numerical range (with respect to some block operator representation). They allow to characterize eigenvalues of self-adjoint operators in gaps of the essential spectrum and eigenvalues of operators which are not self-adjoint in a Hilbert space, but only in a Krein space. From these variational principles, eigenvalue estimates are derived and applied to several examples.

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EQUIVALENCE OF INVERSE STURM-LIOUVILLE PROBLEMS WITH BOUNDARY CONDITIONS RATIONALLY DEPENDENT ON THE EIGENPARAMETER

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The Sturm-Liouville problem $-y''+qy = \lambda y$ with boundary conditions $y(0)\cos\alpha = y'(0)\sin\alpha$ and $[y'/y](1) = [h/g](\lambda)$ is studied, where g and h are real polynomials. Generalised norming constants ρ_n^k associated with eigenvalues Λ_n are defined and formulae are given for the unique recovery of the m-function from these constants. It is then shown that the m-function uniquely determines $\alpha, h/g$ and q and is uniquely determined by by two spectra for different values of α or by the Prüfer angle.

(Research conducted while visiting University of Calgary and University of Saskatchewan and supported in part by the Centre for Applicable Analysis and Number Theory.)

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THE m-FUNCTION FOR A COMPLEX JACOBI MATRIX AND AN APPLICATION TO THE INVERSE RESONANCE PROBLEM

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We study the m-function for a super-exponentially decaying, possibly non-selfadjoint perturbations of the free Jacobi operator and its extension to the nonphysical sheet. A classical theorem of Wiman on the minimum modulus of entire functions allows to control the growth of m on a sequence of circles whose radii tend to infinity. This information is then used to prove that the Jacobi matrix is uniquely determined by the location of all their eigenvalues and resonances up to standard similarity transformations.

This is joint work with B. M. Brown and S. Naboko.

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THE INFLUENCE OF G.G. STOKES ON THE MODERN ASYMPTOTIC THEORY OF DIFFERENTIAL EQUATIONS

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In common with other incumbents of the Lucasian Chair of Mathematics in Cambridge, Sir G.G. Stokes, who held it from 1849 to 1903, was primarily a mathematical physicist. Only 7 of the 109 papers in his five-volume Collected Mathematical and Physical Papers are on mathematical analysis as such, but the present day ramifications of his mathematical papers, especially those of 1857 and 1864 on asymptotic analysis, are out of all proportion to their relatively small number.

In Stokes phenomenon, an asymptotic expansion of a function of a complex variable can change its form near certain rays in the plane (now known as Stokes lines) through the apparently discontinuous appearance of a further series with an exponentially small prefactor and multiplicative constant (the Stokes multiplier). Although small at the place of its birth, this new term can grow to significantly influence the behaviour of the function in other regions.

Because of the absence of exponentially small terms in the subsequently adopted asymptotic definition of Poincare (1886), Stokes phenomenon caused controversy, ambiguity and misunderstanding for over a century, until the physicist M.V. Berry published his insightful paper in 1989. His idea has been developed, using rigorous mathematical analysis, by C.J. Howls, A.B. Olde Daalhuis, F.W.J. Olver and R.B. Paris among others, to provide a firm understanding of the phenomenon. We review the way in which these later authors have made use of two of Stokes's key ideas: the optimal truncation of the dominant series (that is, just before its least term) to give an exponentially small remainder; and secondly the resummation

of the divergent tail of the expansion to improve its computational accuracy. We conclude with an outline of a recently established connection between anti-Stokes lines on the real axis and the absolutely continuous spectrum of differential operators (D.J. Gilbert and A.D. Wood, 2004).

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