Meeting on Computational and Analytic Problems in Spectral Theory

Cardiff School of Computer Science & Informatics, June 6^{th}-9^{th} 2016

Abstract of Talk

DISCRETE DIFFRACTION MANAGED SOLITONS: THRESHOLD PHENOMENA AND RAPID DECAY FOR GENERAL NONLINEARITIES

Dirk Hundertmark
Karlsruhe Institute of Technology, Englerstrasse 2, 76131 Karlsruhe, Germany

We study solitary breather type solutions for a discrete nonlinear Schrödinger type equation. This equation is known to model a wide range of effects ranging from molecular crystals to biophysical systems. Our interest for this equation comes from the fact that it models an array of nonlinear waveguides, the hopping terms corresponding to the interaction of neighbouring waveguides. We consider a variant of this model, the diffraction management equation, where the diffraction is periodically modulated along the waveguides. In experiments stable low power pulses in these waveguide arrays have been observed.

We prove a threshold phenomenon for the existence/non-existence of energy minimizing solitary solutions of the diffraction management equation for strictly positive and zero average diffraction. Our methods allow for a large class of nonlinearities, they are, for example, allowed to change sign, and we impose the weakest possible conditions on the local diffraction profile, it only has to be locally integrable. The solutions are found as minimizers of a nonlinear and nonlocal variational problem which is translation invariant. There exists a critical threshold such that minimizers for this variational problem exist if their power is bigger than the critical threshold and no non-trivial minimizers exist with powers below the critical threshold. We also give simple criteria for the finiteness and strict positivity of the critical threshold. Our proof of existence of minimizers is rather direct and avoids the use of Lions' concentration compactness argument.

Furthermore, we give precise quantitative lower bounds on the exponential decay rate of the diffraction management solitons, which confirm the physical heuristic prediction for the asymptotic decay rate. Moreover, for ground state solutions, these bounds give a quantitative lower bound for the divergence of the exponential decay rate in the limit of vanishing average diffraction. For zero average diffraction, we prove quantitative bounds which show that the solitons decay much faster than exponential. Our results considerably extend and strengthen previous results.

Joint work with: Mi-Ran Choi and Young-Ran Lee, Sogang University, Seoul