# Shape-Aware Line Generalisation With Weighted Effective Area 

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#### Abstract

Few line generalisation algorithms provide explicit control over the style of generalisation that results. In this paper we introduce weighted effective area, a set of area-based metrics for cartographic line generalisation following the bottom-up approach of the Visvalingam-Whyatt algorithm. Various weight factors are used to reflect the flatness, skewness and convexity of the triangle upon which the Visvalingam-Whyatt effective area is computed. Our experimental results indicate these weight factors may provide much greater control over generalisation effects than is possible with the original algorithm. An online web demonstrator for weighted effective area has been set up.


## 1. Introduction

Cartographic line generalisation is one of the major processes in map generalisation. Each cartographic line represents a real world geographic feature. When a map is compiled for a certain purpose from a source map at the same or a greater scale, some "unwanted" details of a cartographic line in the source dataset are eliminated from the representation of the same feature in the target map. In addition, some retained details may be exaggerated to enhance certain characteristics of the feature represented by the line.

### 1.1 Graphic and semantic line generalisation

The main controlling elements in map generalisation include: map purpose and conditions of use, map scale, quality and quantity of data, and graphic limit (Robinson, et al. 1995, pp458). The graphical limitations of the presentation media imply that some details in a line are often not presentable and hence become redundant. Normally the process of eliminating these details is graphically, geometrically and visually driven and will be referred to as graphic line generalisation. On the other hand, detail removal or modification according to the purpose and conditions of use is semantically driven and will be referred to as semantic line generalisation. In the context of this discussion, the term "detail" refers to any three or more consecutive and non-collinear vertices in a cartographic line.

Broadly speaking, graphic line generalisation is very similar to the process normally regarded as line simplification, represented by such algorithms as the Ramer-Douglas-Peucker algorithm (Ramer 1972; Douglas and Peucker 1973). With such a graphically-driven view, the presence of a graphic limit has a two-fold implication: on the one hand, there should be a close association between the tolerances in these algorithms and the graphic limit; on the other hand, it is inappropriate to apply these algorithms with tolerances far beyond (i.e. greater than) the graphic limit. For digital datasets, there is a simple relation between the graphic limit (represented here by presentation resolution $\mathbf{R}$ ) and a database resolution ( $\mathbf{R}_{\mathbf{d b}}$ ), as described in Zhou and Jones (2003):

$$
\begin{equation*}
\mathbf{R}_{\mathrm{db}}=\mathbf{R} / \mathbf{S}=\mathbf{R} * \mathbf{F E} / \mathbf{M E} \tag{1}
\end{equation*}
$$

Here $\mathbf{S}$ is the real scale of the presentation (ratio of $\mathbf{M E}$, the physical extent on the presentation medium, and $\mathbf{F E}$, the corresponding real world extent). Using this relation, graphic generalisation tolerances (normally functions of the database resolution) may be linked to graphic limits for selecting appropriate tolerance values.

Semantic line generalisation is also affected by graphical limitations but mainly controlled by semantic knowledge. To an extent, semantic generalisation is a process of discriminating against some details that are graphically equal to other details in the line. Such a discrimination process could be positive (e.g. a detail below the graphic threshold may be exaggerated and retained) or more frequently, negative (e.g. one of two similar details above the graphic threshold may be eliminated to highlight the other). Note that some effects of semantic generalisation may be achieved by pure graphically driven procedures, albeit unintentionally. Generally speaking, graphic
generalisation tends to retain "extreme" vertices where semantic generalisation tends to eliminate these vertices.

### 1.2 Line generalisation using effective area

Among various (graphic or semantic) line generalisation algorithms, the Visvalingam-Whyatt (1993) algorithm (referred to here as VW algorithm) is of particular interest. This algorithm may be summarised as follows:

- Input: a cartographic line of $n$ vertices (for clarity of illustration, we assume the line is open) with vertices $v_{0}$ and $v_{\mathrm{n}-1}$ as its endpoints.
- Step 1: for each internal vertex $v_{\mathrm{i}}(i=1, n-2)$, calculate its "effective area" $E A_{\mathrm{i}}$ as the area of triangle $v_{\mathrm{i}-1}, v_{\mathrm{i}}$ and $v_{\mathrm{i}+1}$ and assign this value to $v_{\mathrm{i}}$.
- Step 2: Repeat until only $v_{0}$ and $v_{\mathrm{n}-1}$ are left in the line
- Find the vertex $v_{\mathrm{j}}$ with the smallest effective area $E A_{\mathrm{j}}$, temporarily remove it from the line
- Re-calculate effective areas for the two adjacent vertices $v_{j-1}$ and $v_{\mathrm{j}+1}$ as described in step 1 (if the re-calculated area value is smaller than $E A_{\mathrm{j}}, E A_{\mathrm{j}}$ should be assigned to the vertex)
- Step 3: restore all previously removed vertices to the line in their original order
- Output: a cartographic line with all internal vertices labelled with their effective area.

With all vertices processed in this way, given a threshold of minimum effective area, a subset of vertices may be selected to form a less detailed representation of the line.
VW algorithm represents a localised bottom-up approach to line generalisation, using an area-based metric and a fixed detail size of three vertices. The effect of generalisation is gradually propagated from small local details to a more global scale and effects of both graphic and semantic generalisation may be achieved simultaneously. Comparative evaluations of the algorithm are presented in Visvalingam and Williamson (1995) and Visvalingam and Herbert (1999).
In the original form of the algorithm, the shape of the triangle upon which the effective area is measured is not taken into consideration and the simple area value is used directly. Consequently, a very flat triangle and a very tall triangle of the same area (figure 1-A) are treated as having the same effective area and the same cartographic significance. This is, however, not an inherent deficiency of the algorithm. As pointed out in Visvalingam and Whyatt (1993), the essence is the process and any metrics of whatever complexity may be used to generate the effective area.


Figure 1 (A) Two triangles of equal area (B) Triangle Shape Indicators
In the following sections, we present some more sophisticated methods for area computation under which the original form of effective area becomes a special case. We will demonstrate how these methods offer increased control over the generalisation effects.

## 2. Shape Awareness and Weighted Effective Area

A straightforward method to consider the shape of the triangles in effective area computation is to apply a weight factor to the initial effective area. This weight factor will reflect some aspects of the shape of a triangle. Consequently, weighted effective area values are obtained to differentiate triangles with the same area value but of different shape characteristics. By using different weight definitions, different shape characteristics of triangles may be highlighted.

These weight definition functions may be viewed as filters. Filters map the values of some parameters representing the shape of a triangle to a weight factor whose value may be drawn from a finite range (bounded filter) or may be of any non-negative values (un-bounded filter).

For a particular filter, triangles with certain parameter values will have a weight of 1 so that their effective areas are equal to the weighted effective areas under this filter. These triangles are regarded as "standard forms" for this filter.

### 2.1 Some observations on the shape of triangles

For a triangle $T\left(v_{0}, v_{1}, v_{2}\right)$ where $v_{1}$ is the vertex whose effective area is to be calculated (figure 1B), the following parameters may be used to describe the shape characteristics of the triangle (referred to as the effective triangle of $v_{1}$ ):

- Length of base line: $W=\operatorname{Distance}\left(v_{0}, v_{2}\right)$
- Height: $H=\operatorname{Distance}\left(v_{1}\right.$, Line $\left.\left(v_{0}, v_{2}\right)\right)$
- Length of middle line: $M L=\operatorname{Distance}\left(v_{1}, v_{\mathrm{M}}\right)$ where $v_{\mathrm{M}}$ is the middle point of segment $v_{0}-v_{2}$ Using these parameters, we may measure a triangle's flatness (as illustrated in figure 1-A) and skewness (the degree it differs from the isosceles triangle with the same $W$ and $H$ values). In addition, we consider the convexity of a triangle, indicating its orientation relative to a pre-defined vertex order. For example, assuming the three vertices in figure 1-B are part of a closed cartographic line with a counter-clockwise vertex order, if the three-point orientation of $v_{0}-v_{1}-v_{2}$ is counter-clockwise, then triangle $T$ is convex; otherwise, it is concave.


### 2.2 Weighted effective area

For vertex $v_{1}$ in figure 1-B, its original effective area is $\boldsymbol{E A}=0.5 * H * W$ (note that in practice the constant 0.5 may be omitted to improve efficiency). Subsequently, we may define weighted effective area(WEA) as:

$$
\begin{equation*}
W E A=W_{\text {Flat }} * W_{\text {Skew }} * W_{\text {Convex }} * E A \tag{2}
\end{equation*}
$$

where $W_{\text {Flat }} W_{\text {Skew }}$ and $W_{\text {Convex }}$ are flatness, skewness and convexity filter functions respectively.

### 2.3 Flatness filters

Potentially there are numerous ways to define flatness filters. Below are only a few examples we have used in our experiments.

### 2.3.1 High-pass filter (LF)

$W_{\text {flat }}=\left(\frac{4 M \arctan (H /(K S \bullet W)) / P I+N}{M+N}\right)^{K H}$

- Parameters $M>0, N \geq 0$, (controlling maximum range of weight), $K S>0, K H \geq 1$
- For $H /\left(K S^{*} W\right)=1, W_{\text {flat }}=1$ (standard form)
- For $H /\left(K S^{*} W\right)>1, W_{\text {flat }}>1$ (enhancing taller triangles)
- For $H /\left(K S^{*} W\right)<1, W_{\text {flat }}<1$ (weakening flatter triangles)
- For $H \rightarrow \infty, W_{\text {flat }} \rightarrow\left(\frac{2 M+N}{M+N}\right)^{K H}$
- For $H \rightarrow 0, W_{\text {flat }} \rightarrow\left(\frac{N}{M+N}\right)^{K H}$
- For $M=1$ and $N=0, W_{\text {flat }} \in\left[0,2^{K H}\right)$

This filter favours taller triangles and removes flatter triangles. Consequently, extreme points are likely to be retained and the effect is graphic simplification of the line being processed.
2.3.2 Low-Pass filter (HF-01)

$$
\begin{equation*}
W_{\text {flat }}=\left(\frac{4 M \arctan (K S \bullet W / H) / P I+N}{M+N}\right)^{K H} \tag{4}
\end{equation*}
$$

- $M>0, N \geq 0, K S>0, K H \geq 1$
- For $\left(K S^{*} W\right) / H=1, W_{\text {flat }}=1$ (standard form)
- For $\left(K S^{*} W\right) / H>1, W_{\text {flat }}>1$ (enhancing flatter triangles)
- For $H /\left(K S^{*} W\right)<1, W_{\text {flat }}<1$ (weakening taller triangles)
- For $W \rightarrow \infty, W_{\text {flat }} \rightarrow\left(\frac{2 M+N}{M+N}\right)^{K H}$
- For $W \rightarrow 0, W_{\text {flat }} \rightarrow\left(\frac{N}{M+N}\right)^{K H}$
- For $M=1$ and $N=0, W_{\text {flat }} \in\left[0,2^{K H}\right)$

This filter is indeed a symmetric form of LF described above. Thus, it tends to eliminate extreme points and achieves the effect of semantic generalisation.

### 2.4 Skewness filter

For triangle $T\left(v_{0}, v_{1}, v_{2}\right), M L$ is the distance between $v_{1}$ and the middle point of edge $v_{0}-v_{2}$. Consequently, we have the ratio $H / M L \in[0,1]$, which might be used to in a skewness filter:

$$
\begin{equation*}
W_{\text {skew }}=\left(\frac{S M+H / M L}{S M+1}\right)^{S K} \quad(S M \geq 0, S K \geq 1) \tag{5}
\end{equation*}
$$

This filter tends to retain points with effective triangles close to isosceles.
2.5 Convexity filter

$$
\begin{equation*}
W_{\text {convex }}=C \text { (if convex) or } 1 \text { (if concave) } \tag{6}
\end{equation*}
$$

Here $C$ is a positive constant. If $C>1$ is used, this filter tends to retain points with convex effective triangles. Otherwise, points with concave effective triangles are retained.



Figure 2: Sample dataset (and three enlarged details), Crown Copyright 2002

## 3. Experimental Results

To evaluate the generalisation effects of various filters described above, we have used a sample dataset (figure 2) of the coastline of Isle of Wight, which is extracted from an original Ordnance Survey LandForm PANAROMA dataset. There are five (closed) linear objects and 2376 vertices in total in the dataset, where the largest object contains 2236 vertices.

### 3.1 A web demo for weighted effective area

In order to provide a better view on the effects of generalisation based on weighted effective area, we have developed a JAVA applet-based web demonstrator (figure 3) which may be accessed following the link: http://www.cs.cf.ac.uk/user/S.Zhou/


Figure 3: Web Demo for weighted effective area filters
This demonstrator allows comparison of various generalisation results using weighted effective area to that of RDP and the original Visvalingam-Whyatt algorithm, where parameter values (i.e. RDP tolerance, effective area or weighted effective area) or the number of vertices retained after generalisation are adjustable.
In the following subsection, we will present a few experimental results that demonstrate the different effects of various generalisation filters described above. These results are obtained from the web demonstrator. For each comparison, the same number of filtered vertices is retained in the whole generalised dataset so that the difference in vertex selection/filtering of various algorithms or parameter values may be highlighted. Also in all experiments $M=0$ and $N=1$.
3.2 Experiments


Figure 4: Effects of VW with Skewness (D) compared to original (A), RDP(B) and VW (C) (for B,C and D, 1000 vertices retained in the whole dataset)

### 3.2.1 Skewness

According to our current experimental results, the weight based on the skewness of the effective triangle does not make a significant impact on the output if moderate parameter values are used (e.g. figure 4-D, $S M=0$ and $S K=2$ ). On the other hand, more extreme parameter values may generate unpredictable and undesirable results. These results cast doubt on the value of using this weight factor.

### 3.2.2 Convexity

In our experiments, extreme convexity weight values generate quite significant effects (figure 5). On top of the initial effective area value, application of a very small weight ( $C=0.004$ ) tends to retain vertices on the external local convex hulls while a very large weight ( $C=25$ ) has the opposite effect of retaining vertices on the internal local convex hulls (see Normant and van de Walle 1996, regarding local convex hulls).



Figure 5: Effects of applying convexity weight to VW
(1000 vertices retained in the whole dataset)
Figure 6 shows the result of combining a convexity filter ( $C=26$ ) and a low-pass flatness filter (HF01+Convex). For purposes of comparison, results of RDP, VW and the flatness filter alone (HF01) are shown.





Figure 6: Effects of Convex filter (C=26, 150 vertices retained)

### 3.2.3 Line simplification with WEA - LF Scheme

Figure 7 and 8 demonstrate the effect of graphic simplification using "high-pass" weighted effective area, in comparison to the results of RDP. This filter appears to be able to generate simplification effects similar to that of RDP.





Figure 7: Simplification by RDP, $\mathrm{LF}(K S=0.5, K H=1)$ and $\mathrm{LF}(K S=1, K H=1)$ (1000 vertices retained in the whole dataset)




Figure 8: Simplification by RDP and $\operatorname{LF}(K S=0.2, K H=1)$, 200 vertices retained

### 3.2.4 Line generalisation with WEA - HF01

The effects of the low-pass filter HF01 are shown in figures 9-12. Figures 9 and 10 demonstrate the effect of defining different "standard forms" (i.e. weight equals to 1 ), represented by $K S$ ( $K S=0.2$, 0.5 and 1). Clearly, a "flatter" standard form (i.e. with a smaller $K S$ ) results in heavier generalisation.


Figure 9: Effects of HF01 (detail 1) - Original VW, KS/KH as: $0.2 / 1 ; 0.5 / 1 ; 1 / 1$ (1000 vertices retained in the whole dataset)


Figure 10: Effects of HF01 (detail 2) - Original VW, KS/KH as: 0.2/1; 0.5/1; 1/1
Figure 11 shows the generalisation effects of the same set of parameter values at different levels of detail (vertices retained in the whole dataset are from 1000 to 150).


Figure 11: Effects of HF01 at $\mathrm{KS}=0.5$ \& $\mathrm{KH}=2$ with 1000/600/300/150 vertices (1-4) retained
Finally, figure 12 illustrates the effect of different values for KH . It is obvious that at the same level of details ( 1000 vertices), a larger KH value will result in heavier generalisation.


Figure 12: Effects of different KH (1/2/4/8) for the same KS (1), 1000 vertices

## 4. Discussion

The experimental results in the previous section have demonstrated that the application of weight factors for flatness, convexity and (to a lesser extent) skewness can provide considerable control over both graphic and semantic generalisation effects. Apart from the skewness filter, the filter parameters provide consistent and predictable control over the resulting generalisation.

It is worth noting that greater control does not always result in a "better" effect, but the parameters described and demonstrated here do appear to provide excellent potential for obtaining generalisations that are adapted to the requirements of particular applications. At the current stage of research and development, we suggest WEA may best be applied in an interactive manner in order to obtain preferred results, for which an interactive mapping tool has been provided. In future it may be possible to select parameter values automatically based on the results of training with different types of generalisation.

### 4.1 Topologically consistent generalisation

A problem associated with VW algorithm (as well as many other algorithms such as RDP) is that topological consistency is not guaranteed. WEA-based generalisation is no exception as the same bottom-up process as in VW algorithm is adopted. It is however fairly easy to geometrically (i.e. graphically) remove inconsistencies by adopting simple approaches such as retaining any vertex whose removal may cause inconsistency. For example, the topologically consistent multirepresentational dataset used in (Zhou and Jones 2003) is generated in this way. There a Delaunay triangulation was used to detect when removal of a vertex would cause an inconsistency.

### 4.2 The issue of feature partition

As mentioned earlier, the bottom-up process in VW algorithm is a localised, minimalist approach as only the smallest details (three vertices) are considered. Generalisation effects on larger details are achieved progressively without explicit knowledge of them. The lack of direct control over these (often semantically significant) details makes it more difficult to decide the best combination of parameter values. Indeed, often a single set of parameters may not be appropriate for every large detail in the cartographic line to be generalised (which is especially true for the convexity filters). Therefore, it is natural to consider partitioning the line into several large details and subsequently applying bottom-up or other types of generalisations on them with appropriate individual parameter sets.

Many methods for (geometric or semantic) detail identification and feature partition have been proposed, such as Sinuosity measures (Plazanet 1995), Voronoi Skeletons (Ogniewicz and Kübler 1995), skeletons based on Delaunay triangulation (van der Poorten and Jones 2002) and various convex hull based methods (e.g. Normant and van de Walle 1996; Zhou and Jones 2001). Following partitioning of features and addressing issues such as hierarchical details, overlapped details and oriented details of larger scale (i.e. more vertices), the best overall generalisation effects may be achieved by combining a localised bottom-up generalisation method, as presented here, with methods which take a more global view of features and operate successfully at the level of larger details (such as the branch pruning approach of van der Poorten and Jones, 2002) or multiple features.

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