LIMIT SPECTRAL CURVES FOR THE ORR-SOMMERFELD OPERATOR AND QUASICLASSICAL EIGENVALUE DISTRIBUTION ALONG THESE CURVES

A A SHKALIKOV
ashkalikov@yahoo.com
Department of Mechanics and Mathematics
Moscow Lomonosov State University
Moscow, 198 899, Russia

We study the well-known in hydrodynamics Orr-Sommerfeld problem on the finite interval $[-1, 1]$. We are interested in the question: how do the λ–eigenvalues of this problem behave as the Reynolds number $R \to \infty$? To answer this question we consider an auxillary model problem

$$i\varepsilon^2 y'' + q(x)y = \lambda y,$$
$$y(-1) = y(1) = 0,$$

where $\varepsilon = (\alpha R)^{-1/2}$. The limit behaviour of the eigenvalues of this problem as $\varepsilon \to 0$ depends on analytic properties of $q(z)$, namely, on the topology the Stokes lines, which are defined in the complex plane by the equation

$$\operatorname{Re} S(z) = 0, \qquad S(z) = \int_{\xi_\lambda}^{1} \sqrt{i\,(q(\xi) - \lambda)}\, d\xi,$$

where ξ_λ is the root of $q(\xi) - \lambda = 0$.

We investigate the model problem in two cases: 1) an analytic monotone profile $q(z)$, 2) the so–called Couette–Poiseuille profile $q(z) = az^2 + bz + c,\;\; a, b, c \in \mathbb{R}$. We prove that the spectrum of the model problem is concentrated along some critical curves in the complex plane, defined by the Stokes lines, which for a monotone profile form a "spectral tie". We find the formulas for the eigenvalue distribution along the limit curves which are uniform with respect to ε. Then we prove that the limit spectral curves for the original Orr-Sommerfeld problem coincide with those for the model problem, as well as the main terms in the formulas for the eigenvalue distribution along the curves.