A Meeting to Mark the Sixty-fifth Birthday of Professor M S P Eastham

26th-27th July, 2002

University of Wales
Gregynog Hall
Newtown
Powys

London --------------------------------- Cardiff University
Mathematical Society --------------------------------- Department of Computer Science
A Meeting to Mark the Sixty-fifth Birthday of Professor M S P Eastham

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Michael Eastham received his early education in Manchester Grammar School and Mer-
ton College, Oxford. In 1959, after a successful undergraduate career, he was awarded
a Domus Senior Scholarship by Merton College to enable him to pursue postgraduate
studies under the guidance of E.C.Titchmarsh. In 1961, in recognition of the excellence
of the research for his D.Phil thesis, he was presented with the prestigious Senior Math-
ematical Scholarship by the University of Oxford. In 1978 he was awarded the Keith
Prize and Gold Medal by the Royal Society of Edinburgh for his contribution to classical
analysis, and was made a Fellow of the Royal Society of Edinburgh in 1982.]

After a year as Junior Research Fellow of Merton College, Michael left Oxford in 1962
to take up an appointment at Reading University. Later he held positions in the uni-
versities of Southampton, London (Chelsea College and King’s College) where he was
Professor of Pure Mathematics 1980-1988, and Bahrain. At present he is an honorary
Professorial Fellow of the University of Wales at Cardiff.

He has made a significant contribution to classical analysis, with an impressive output
of 4 books and over 100 papers to date. His main area of expertise is the spectral the-
ory of linear ordinary differential equations. Of notable significance is his work on the
asymptotics of solutions. Using the technique of repeated diagonalisation with consum-
mate skill and efficacy, he has obtained powerful results on asymptotics in a number of
papers which were the basis of his well-known book on the subject. His results have
enabled him and his students to make a comprehensive analysis of the deficiency index
problem for equations with suitably smooth coefficients. In recent years he has devel-
oped his work on asymptotics in a manner suitable for inclusion as an algorithm that can
be realised on a computer, and has applied his methods effectively to a problem of con-
siderable current interest, namely, that of the location of Sturm-Liouville resonances.
Out of Michael’s long and impressive list of publications, the papers listed below are
particularly noteworthy, and are good examples of the power of his analysis.

Michael has been a leading figure in the subject for the last forty years. He has lec-
tured on his work in many countries, and his books have become standard texts. He
continues to be very active in research, his current main areas of activity being spectral
concentration and resonance problems.

SELECTED PUBLICATIONS

1. B. M. Brown and M. S. P. Eastham. Analytic continuation and resonance-free

This Laudatum will appear in a future edition of the Journal of Computational and Applied Mathe-
ematics.


SPEAKERS

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TIMETABLE OF TALKS AND EVENTS

FRIDAY, JULY 26TH, 2002

Chair  W D Evans

1500  E B Davies  Bounds on the eigenvalues of non-self-adjoint Schrödinger operators

1600  Tea

1630-1730  W N Everitt† and Anthippi Poulkou  First-order linear boundary value problems

1900  Reception

1930  Dinner

followed by the Laudatum by Prof W N Everitt.

SATURDAY, JULY 27TH, 2002

0800  Breakfast

Chair  B M Brown

0900  A A Balinsky and W D Evans†  On the spectral properties of the Brown-Ravenhall operator

1000  Hubert Kalf  On the spectral theory of Dirac operators with a variable mass term

1100  Coffee

1130  J B McLeod  Stability of Poiseuille Flow

1230  Lunch

Depart

Where there is more than one author, the speaker is marked thus †.
ABSTRACTS OF PRESENTATIONS

BOUNDS ON THE EIGENVALUES OF NON-SELF-ADJOINT SCHröDINGER OPERATORS

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This talk describes bounds on the location of the eigenvalues of one-dimensional non-self-adjoint Schrödinger operators, obtained jointly by the speaker and J Nath. The bounds are expressed in terms of $L^p$ norms of the potentials, and are optimal if the potential is $L^1$.

ON THE SPECTRAL PROPERTIES OF THE BROWN-RAVENHALL OPERATOR

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The fact that the Dirac is unbounded below creates problems if it is used to describe multi-particle relativistic systems since the resulting operator has a spectrum which covers the whole of the real line. To overcome this difficulty Brown and Ravenhall proposed the following one-particle model. To describe an electron in the field of its nucleus and subject to relativistic effects, the operator of Brown and Ravenhall is

\[(1) \quad B := \Lambda_+ \left( D_0 - \frac{e^2 Z}{|\cdot|} \right) \Lambda_+.\]

acting in the Hilbert space $\mathcal{H} := \Lambda_+(L^2(\mathbb{R}^3) \otimes \mathbb{C}^4)$. The notation in (1) is as follows
• $D_0$ is the free Dirac operator

$$D_0 = c\alpha \frac{\hbar}{i} \nabla + mc^2\beta \equiv \sum_{j=1}^{3} c \frac{\hbar}{i}\alpha_j \frac{\partial}{\partial x_j} + mc^2\beta,$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and $\beta$ are the Dirac matrices given by

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

with $0_2, 1_2$ the zero and unit $2 \times 2$ matrices respectively and $\sigma_j$ the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

• $\Lambda_+$ denotes the projection of $L^2(\mathbb{R}^3) \otimes \mathbb{C}^4$ onto the positive spectral subspace of $D_0$, that is $\chi_{(0,\infty)}(D_0)$, where $\chi_{(0,\infty)}$ is the characteristic function of $(0, \infty)$. If we set

$$\hat{f}(p) \equiv \mathcal{F}(f)(p) = \left(\frac{1}{2\pi\hbar}\right)^{3/2} \int_{\mathbb{R}^3} e^{-i\mathbf{x} \cdot \mathbf{p}/\hbar} f(\mathbf{x}) \, d\mathbf{x}$$

for the Fourier transform of $f$, then it follows that

$$(\Lambda_+ f)^\wedge(p) = \Lambda_+(p)\hat{f}(p),$$

where

$$\Lambda_+(p) = \frac{1}{2} + \frac{c\alpha \cdot \mathbf{p} + mc^2\beta}{2e(p)}, \quad e(p) = \sqrt{c^2p^2 + m^2c^4}$$

with $p = |\mathbf{p}|$.

• $2\pi\hbar$ is Planck’s constant, $c$ the velocity of light, $m$ the electron mass, $-e$ the electron charge, and $Z$ the nuclear charge.

The lecture will discuss spectral properties of operators $b_{l,s}$ appearing in the partial wave decomposition of $B$: the indices $l, s$, denote the angular momentum channel and spin respectively. The following topics will be covered: the value of the critical charge $Z_c(l, s)$ which yields the positivity of $b_{l,s}$, the charge range for essential self-adjointness, and the charge range for the absence of embedded eigenvalues.
1. Abstract

This lecture reports on joint work with Anthippi Poulkou, Department of Mathematics, University of Athens.

The general Lagrange symmetric first-order differential equation with Lebesgue integrable coefficients, on the open interval \((a, b)\) of the real line \(\mathbb{R}\), has the form, defining the differential expression \(M[\cdot]\),

\[
M[y](x) := i\rho(x)y'(x) + \frac{1}{2}i\rho'(x)y(x) + q(x)y(x) = \lambda w(x)y(x) \quad \text{for all } x \in (a, b)
\]

where \(\lambda \in \mathbb{C}\) is the complex spectral parameter. Here the coefficients \(\rho, q, w\) satisfy the conditions

\begin{enumerate}
  \item \(\rho, q, w : (a, b) \to \mathbb{R}\)
  \item \(\rho \in AC_{\text{loc}}(a, b)\) and \(\rho(x) > 0\) for all \(x \in (a, b)\)
  \item \(q, w \in L_{\text{loc}}^1(a, b)\)
  \item \(w(x) > 0\) for almost all \(x \in (a, b)\).
\end{enumerate}

The right-definite spectral analysis for this differential equation takes place in the Hilbert function space \(L^2((a, b); w)\) with norm and inner-product

\[
\|f\|_w^2 := \int_a^b w(x) |f(x)|^2 \quad \text{and} \quad (f, g)_w := \int_a^b w(x)f(x)\overline{g(x)} \, dx.
\]

A necessary and sufficient condition to ensure that the differential expression \(M[\cdot]\) generates a maximal operator in \(L^2((a, b); w)\) with equal deficiency indices \(d^\pm = 1\) whose self-adjoint restrictions have discrete spectra, is

\[
\int_a^b w(x) \rho(x) \, dx < +\infty.
\]

With this condition satisfied the GKN boundary condition method can be applied to give symmetric boundary value problems with the following properties:
Theorem 1.1. Let $T$ be a self-adjoint restriction of the maximal operator generated by $M[·]$; then $T$ has the following spectral properties:

(i) The spectrum $\sigma(T)$ of $T$ in $L^2((a, b); w)$ is simple and discrete.

(ii) The spectrum $\sigma(T)$ is unbounded above and below on $\mathbb{R} \subset \mathbb{C}$, and so may be denoted by, here $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots \}$,

$$\sigma(T) = \{\lambda_n \in \mathbb{R} : n \in \mathbb{Z}\}$$

with

$$\lambda_n < \lambda_{n+1} \text{ for all } n \in \mathbb{Z}, \text{ and } \lim_{n \to \pm \infty} \lambda_n = \pm \infty.$$ 

(iii) There exists a positive number $k > 0$, with

$$k = 2\pi \left( \int_a^b \frac{w(x)}{\rho(x)} \, dx \right)^{-1},$$

such that

$$\lambda_{n+1} - \lambda_n = k \text{ for all } n \in \mathbb{Z}.$$ 

(iv) There exists an entire (integral) function $\varphi : \mathbb{C} \to \mathbb{C}$, generated by the boundary value problem, with the properties

(i) $\varphi(\lambda) = 0$ if and only if $\lambda \in \{\lambda_n : n \in \mathbb{Z}\}$

(ii) $\varphi'(\lambda_n) \neq 0$ for all $n \in \mathbb{Z}$.

2. KRAMER ANALYTIC KERNELS

The boundary value problems discussed in Section 1 generate Kramer analytic kernels in the Hilbert space $L^2((a, b); w)$.

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References


ON THE SPECTRAL THEORY OF DIRAC OPERATORS WITH A VARIABLE MASS TERM

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The spectrum of the Dirac operator is purely discrete when the mass term “dominates” the potential (O. Yamada). In the opposite case one expects the spectrum to be purely absolutely continuous. This was proved when both the mass term and the potential are spherically symmetric (K. M. Schmidt, O. Yamada). Using virial techniques, a theorem is presented which establishes at least absence of eigenvalues when mass term and potential are not necessarily rotationally symmetric. This is joint work with T. Okaji (Kyoto) and O. Yamada (Kusatsu).

STABILITY OF POISEUILLE FLOW

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Poiseuille flow is two-dimensional flow in a straight pipe. The question of the stability of the steady flow, particularly with a parabolic velocity profile, is a long-standing one that has received intense treatment numerically, but very little analytically. The talk will examine what we can prove about this problem.
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