TWO METHODS FOR SPECTRALLY CORRECT APPROXIMATION OF MAXWELL CAVITY EIGENFREQUENCIES: WEIGHTED REGULARIZATION AND DISCRETE COMMUTING DIAGRAMS

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Spectrally correct approximation of Maxwell cavity eigenfrequencies is an issue because the curl.curl operator has an infinite dimensional kernel formed of potential fields (gradients). There are two families of methods which can overcome this unfavourable feature: (i) The regularization for which the curl.curl non zero eigenvalues are unchanged, while the zero eigenvalues are transformed in an infinite sequence of positive eigenvalues which tend to infinity, the whole spectrum being computed by standard FEM (ii) Special FEM based on a discrete commuting diagram, where the kernel is approached by discrete gradients, hence the zero eigenvalues stay at zero.

Each of these methods encounters its own difficulty.

(i) The FEM approximation of the regularized equation produces wrong results if the domain has non-convex corners or edges. We will present the *weighted regularization* of [1] which is a modification of the standard method, and restores a correct approximation.

(ii) Because methods based on discrete commuting diagrams do not provide divergence-free solutions, and because the infinite dimensional kernel is partly approached by each discretization, the proof of the spectral correctness requires, as shown by Kikuchi, checking a special property called the *discrete compactness*. This is something which is not straightforward to prove for actual families of finite elements. We present the recent paper [2] where the discrete compactness is proved for the p-version of finite elements in a general framework for the first time, taking advantage of the regularizing properties of a newly analyzed Poincaré operator [3].

References

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[3] M. Costabel, A. McIntosh: On Bogovski and regularized Poincaré integral operators and the de Rham complex without boundary conditions on Lipschitz domains. In preparation, 2008.