STABILITY ESTIMATES FOR THE INVERSE CONDUCTIVITY PROBLEM

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We consider the problem

$$\begin{aligned} \operatorname{div}(\gamma \nabla u) &= 0 \quad \text{in } \Omega \\ u &= \varphi \quad \text{on } \partial\Omega, \end{aligned}$$

where $\gamma > 0$ is the conductivity parameter. We assume that the Dirichlet-to-Neumann map Λ_γ is given on some open part $\Gamma \subset \partial\Omega$ of the boundary. That means that given φ on $\partial\Omega$ we know $\gamma \partial_n u|_\Gamma$.

We are interested in the inverse problem which asks for γ if Λ_γ is known. Our aim is to prove stability estimates for γ in terms of a suitable operator norm of Λ_γ.

In the presentation we will analyze two situations and give stability estimates for the corresponding Dirichlet-to-Neumann map. On the one hand we will assume that γ is smooth ($\gamma \in C^2$) but the Neumann data is given on a part of the boundary only. The stability estimates we get in this case are of log-log-type. By the work of Alessandrini it is known that a log-type stability estimate holds in the case of full boundary data. Furthermore, Mandache proved that in general this estimate is optimal. If the domain Ω has a special structure we also get a log-type estimate in the partial data case.

As a second situation we will discuss the case where $\gamma \in C^{3/2+\varepsilon}$ and the Neumann data is known on the whole boundary. In this case we prove log-type stability estimates. Since the complex geometrical optics solutions cannot be constructed by transforming the equation to a Schrödinger equation, we will approximate the conductivity γ first.