ON SPECTRAL PROPERTIES OF NONSELF-ADJOINT OPERATORS WITH ALMOST HERMITIAN SPECTRUM

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The class of operators with almost Hermitian spectrum is quite a natural choice if one wishes to move on from the study of dissipative operators to the general situation of non-dissipative ones. It turns out, that the possibility of appearance of the spectral subspace \( N^0_i \) separates these two cases, at least in the situation of relatively trace class perturbations. We call an operator in a Hilbert space \( H \) satisfying the condition \( H = N^0 \) an operator with almost Hermitian spectrum. Detailed investigation of the spectral component \( N^0 \) reveals that its properties closely resemble those of self-adjoint operators with purely singular spectrum, hence the name “almost Hermitian”. We present some recent results obtained in this area, paying special attention to the so-called matrix model case (the case of a rank two completely nonself-adjoint perturbation of a self-adjoint operator). The latter constitutes a rather transparent model situation, already revealing all the major difficulties to be found in the general case. In particular, we discuss the almost-spectral theorem for the class of operators described above, thus establishing yet another link between operators with almost Hermitian spectrum and singular self-adjoint operators.