A-PRIORI BOUNDS FOR DISCRETE AND CONTINUOUS BOUNDARY VALUE PROBLEMS

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For a model problem involving the Laplace operator

$$(*) \qquad -\Delta u = f(x,u) \text{ in } \Omega \subset \mathbb{R}^n, \qquad u = 0 \text{ on } \partial\Omega$$

where the non-linear right-hand side has a power-like growth behavior, e.g., $f(x,s) = (1+|s|^p)$, we explain three different types of solutions: **classical, weak** and **very weak**. One of the main questions for $(*)$ is the question of a-priori bounds versus singularities. It turns out that three different aspects determine whether positive solutions of $(*)$ satisfy a-priori bounds or develop singularities:

- the solution concept,
- the growth rate p of the nonlinearity $f(x,s)$,
- the regularity of Ω.

A typical results reads as follows: for $1 < p < p^*$ all positive solutions of $(*)$ are uniformly bounded whereas for $p > p^*$ a-priori bounds fail and sequences of solutions forming singularities exist. For classical and weak solutions the critical exponent $p^* = \frac{n+2}{n-2}$ is independent of the geometry of Ω whereas for very-weak solutions the critical exponent is typically smaller than $\frac{n+2}{n-2}$ and does depend explicitely on the geometry of Ω.

Moreover, if $(*)$ is discretized via a finite difference method one can ask if the discretized problem has the same critical exponent as the continuous problem. It turns out that an affirmative answer can be given with help of the very weak solution-concept.

In the talk I will report results obtained in collaborations with J. Horak, P.J. McKenna, P. Quittner and T. Weth.