

# An Overview of Labelling-Based Justification Status

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# Preliminaries

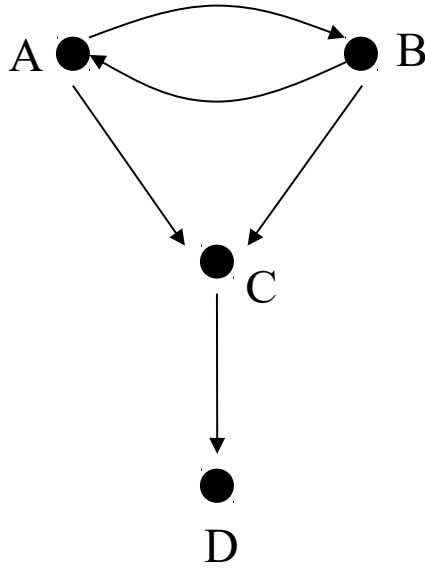
- *Argumentation framework*: graph  $(Ar, att)$  in which
  - the nodes  $(Ar)$  represent a given set of *arguments*,
  - the arrows  $(att)$  represent the *attack* relation.
- A *labelling* is a function  $Lab: Ar \rightarrow \{\text{in}, \text{out}, \text{undec}\}$ .
- A *complete labelling* is a labelling s.t. For each argument  $A$ ,
  - $A$  is labelled **in** iff all its attackers are labelling **out**.
  - $A$  is labelled **out** iff it has an attacker that is labelled **in**.

eg. in a gun fight

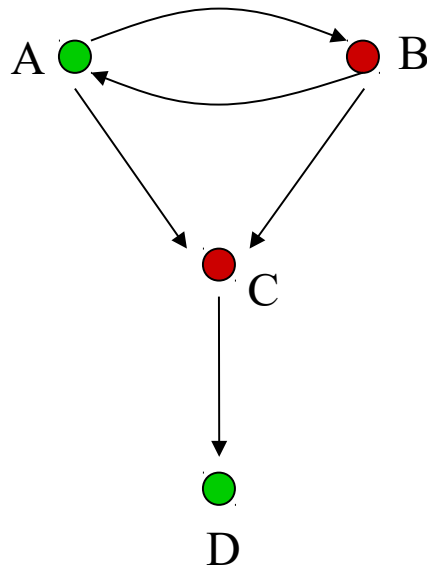
You survive iff all your attackers are killed.

You get killed iff at least one attacker remains alive.

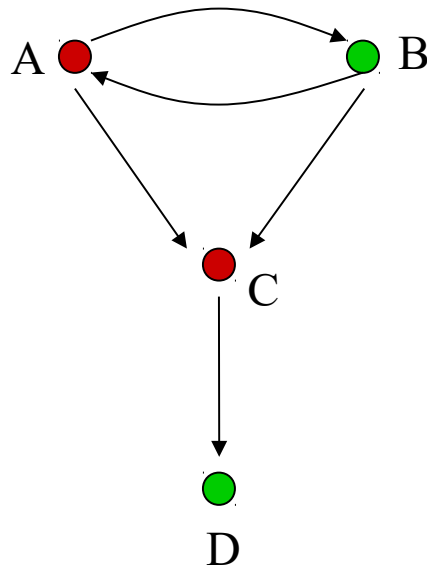
# An Example



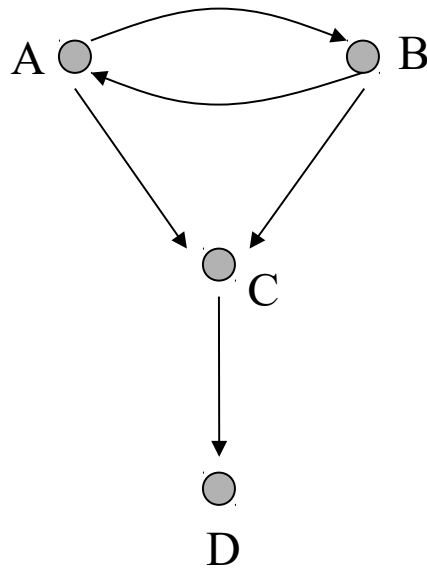
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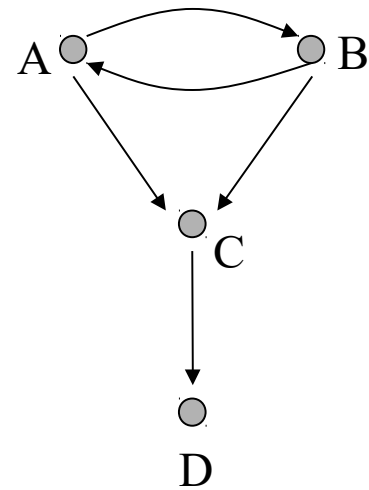
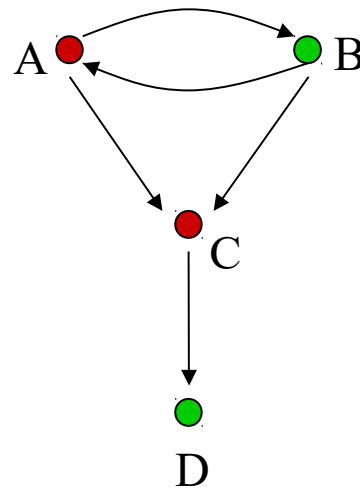
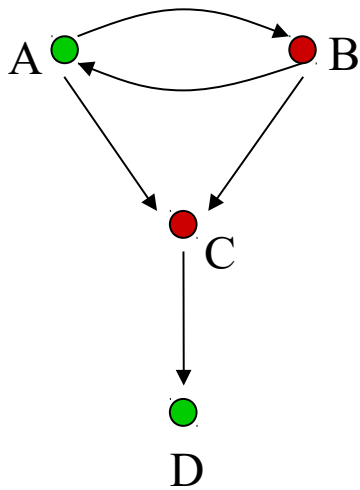


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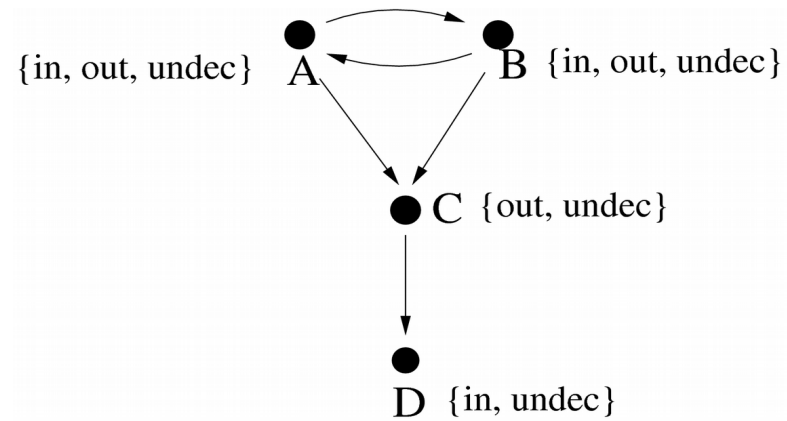
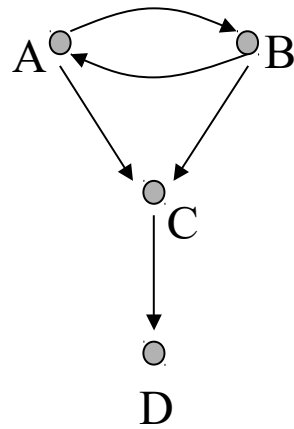
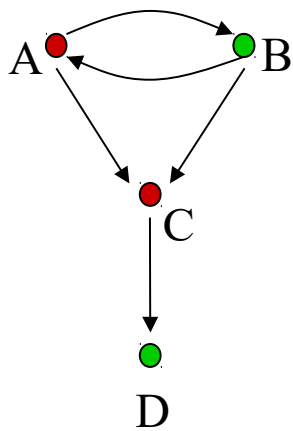
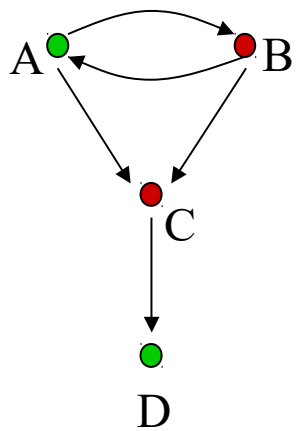
# Justification Status

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# Justification status

$\mathcal{JS}(A) =$

$\{\text{in}\}$  iff  $A$  is in every complete extension.

$\{\text{out}\}$  iff  $A$  is attacked by every complete extension.

$\{\text{undec}\}$  iff  $A$  is not in any complete extension and  
 $A$  is not attacked by any complete extension.

$\{\text{in, undec}\}$  iff  $A$  is not in a complete extension and  
 $A$  is in a complete extension and  
no complete extension attacks  $A$ .

$\{\text{out, undec}\}$  iff  $A$  is not in any complete extension and  
 $A$  is attacked by a complete extension and  
there is a complete extension does not attack  $A$ .

$\{\text{in, out, undec}\}$  iff  $A$  is in a complete extension and  
 $A$  is attacked by a complete extension.

$\{\text{in, out}\}$ : impossible

$\emptyset$ : impossible

# Justification status

$\mathcal{JS}(A) =$

$\{\text{in}\}$  iff  $A$  is in the grounded extension.

$\{\text{out}\}$  iff  $A$  is attacked by the grounded extension.

$\{\text{undec}\}$  iff  $A$  is not in any admissible extension and  
 $A$  is not attacked by any admissible extension.

$\{\text{in, undec}\}$  iff  $A$  is not in the grounded extension and  
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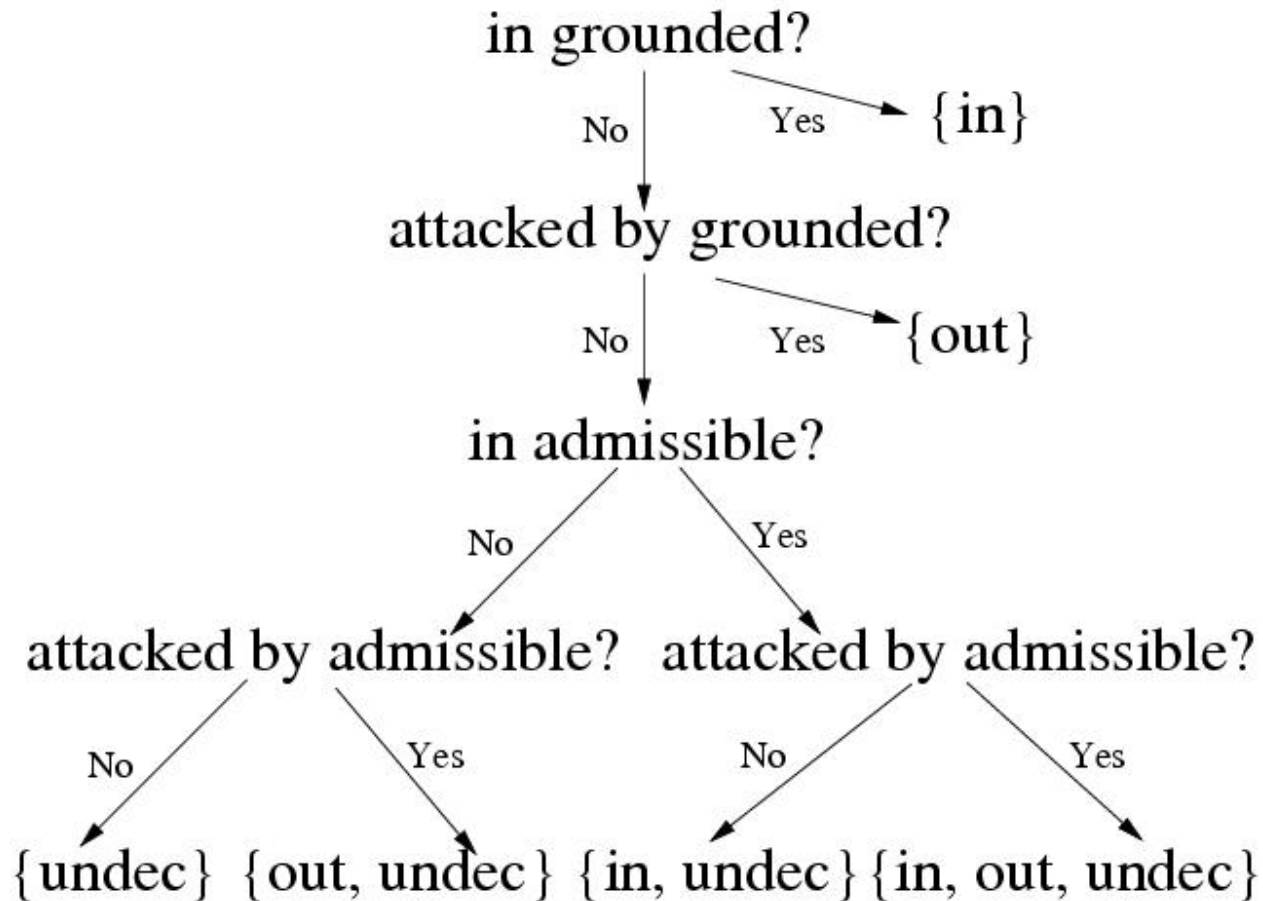
$\{\text{out, undec}\}$  iff  $A$  is not in any admissible extension and  
 $A$  is attacked by an admissible extension and  
 $A$  is not attacked by the grounded extension.

$\{\text{in, out, undec}\}$  iff  $A$  is in an admissible extension and  
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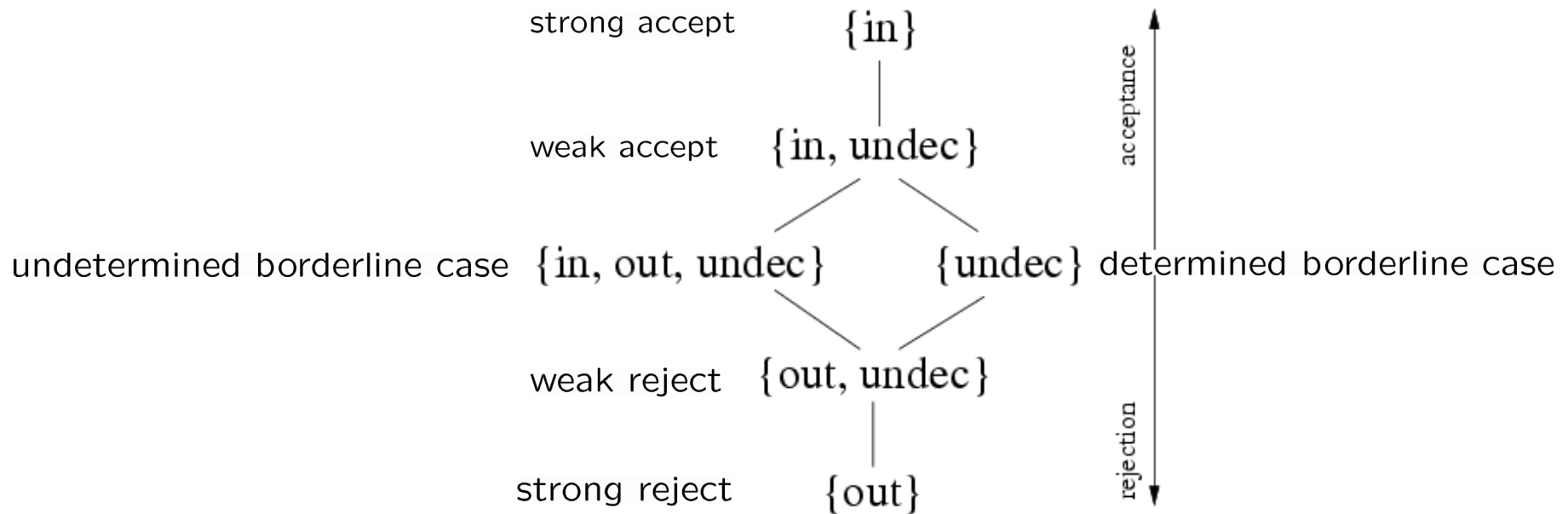
$\{\text{in, out}\}$ : impossible

$\emptyset$ : impossible

# Justification status



# Degrees of Justification status



# Justification status

## Proposition

- $A$  is in the grounded extension iff it is strongly accepted.
- if  $A$  is in every preferred extension then  $A$  is strongly or weakly accepted.
- $A$  is in at least one preferred extension iff  $A$  is strongly accepted, weakly accepted, or undetermined borderline.
- if  $A$  is strongly accepted then  $A$  is in every semi-stable extension.  
if  $A$  is weakly accepted then  $A$  is in at least one semi-stable extension.
- $A$  is in an ideal set iff  $A$  is member of an admissible set consisting only of strongly or weakly accepted arguments.

# Justification Status of Conclusions

- each argument  $A$  has a conclusion  $\text{Conc}(A) \in \mathcal{L}$
- a conclusion labelling is a function  
 $\text{ConcLab}: \mathcal{L} \rightarrow \{\text{in}, \text{out}, \text{undec}\}$
- Given an argument labelling  $\text{ArgLab}$ , we define the associated conclusion labelling  $\text{ConcLab}$  s.t.  $\text{ConcLab}(c)$  is the label of the “best” argument for  $c$  (or out, if no argument for  $c$  exists)

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 $\text{ConcLab}(c) = \max(\{\text{ArgLab}(A) \mid \text{Conc}(A)=c\} \cup \{\text{out}\})$
- $\text{JS}(c) = \{\text{ConcLab}(c) \mid \text{ConcLab} \text{ is a complete conclusion labelling}\}$



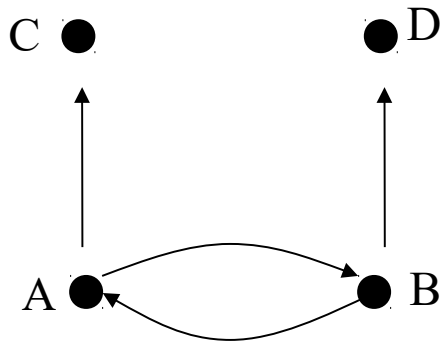
# Example: Dealing with Floating Conclusions

- Brygt Rykkje is Dutch because he was born in Holland
- Brygt Rykkje is Norwegian because he has a Norwegian name
- Brygt Rykkje likes ice skating because he is Norwegian
- Brygt Rykkje likes ice skating because he is Dutch

# Example: Dealing with Floating Conclusions

- John says the suspect killed the victim by stabbing him
- Bob says the suspect killed the victim by shooting him
- The suspect killed the victim,  
because Bob says the suspect killed the victim by shooting him
- The suspect killed the victim,  
because John says the suspect killed the victim by stabbing him

# Example: Dealing with Floating Conclusions



$\text{Conc}(A) = a$

$\text{Conc}(B) = b$

$\text{Conc}(C) = e$

$\text{Conc}(D) = e$

$\text{ArgLab}_1 = (\{A, D\}, \{B, C\}, \emptyset)$

$\text{ConcLab}_1 = (\{a, e\}, \{b\}, \emptyset)$

$\text{ArgLab}_2 = (\{B, C\}, \{A, D\}, \emptyset)$

$\text{ConcLab}_2 = (\{b, e\}, \{a\}, \emptyset)$

$\text{ArgLab}_3 = (\emptyset, \emptyset, \{A, B, C, D\})$

$\text{ConcLab}_3 = (\emptyset, \emptyset, \{a, b, e\})$

$\text{JS}(e) = \{\text{in}, \text{undec}\}$  (*weak accept*)

# Labelling-Based JS:

- provides *levels* of justification based on *standard* AFs (so no probabilities or other numerical add-ons)
- provides a more *refined* status than the usual extension based approach (e.g. grounded or credulous preferred)
- can easily be *computed* (based on existing proof procedures for grounded and preferred)
- can be applied to *arguments* as well as to *conclusions* (floating conclusions become weakly accepted)

# Literature

- Yining Wu and Martin Caminada  
*A Labelling-Based Justification Status of Arguments*  
Studies in Logic 3(4):12-29 (2010)
- Wolfgang Dvořák  
*On the Complexity of Computing  
the Justification Status of an Argument*  
*TAF A post proceedings, pages 32-49 (2012)*