New Trends in Spectral Theory and Applications

Cardiff, UK, 18th-20th December 2010

This meeting celebrates the 80th birthday of Professor D E Edmunds and the 70th birthday of Professor W D Evans

Inverse spectral and scattering theory associated to the Camassa-Holm equation

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The Schrödinger equation is of crucial importance to the theory of the KdV equation via its scattering/inverse scattering theory. In the same way, the Camassa-Holm equation is linked with scattering/inverse scattering theory for the equation

$$-u'' + \frac{1}{4}u = \lambda w u.$$

Here no inverse scattering is available unless $w \geq 0$ and w is smooth. There is interest in the case when w changes sign. In this talk we shall discuss some work in progress on scattering / inverse scattering theory for this equation when w is allowed to change sign, both on $\mathbf{R}^+$ and on $\mathbf{R}$.

Joint work with C Bennewitz (Lund), R Weikard (Birmingham AL).

GRADIENT REGULARITY IN ELLITPIC PDE'S VIA REARRANGEMENTS

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We derive sharp regularity estimates for the gradient of solutions to boundary value problems for p-Laplacian type equations. Domains with minimal regularity are allowed. Our approach relies upon an analysis on the level sets of the modulus of the gradient, and on ensuing rearrangement estimates for the latter. This talk is based on a joint work with V.Maz'ya

ENTROPY NUMBERS, MEASURE OF NON-COMPACTNESS AND INTERPOLATION

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The close connection between entropy numbers of a compact operator and its eigenvalues, has motivated an intensive study of the asymptotic behaviour of entropy numbers of embeddings between function spaces. In this research, interpolation properties of entropy numbers are very useful tools.

We shall revise these properties as well as the interpolation properties of the measure of non-compactness. We consider not only the real method $(A_0, A_1)_{\theta,q}$ and the complex method $[A_0, A_1]_\theta$ with $0 < \theta < 1$ and $1 \leq q \leq \infty$, but also the limiting real methods that come up by the choice $\theta = 0$ and $\theta = 1$.

MUCKENHOUPT WEIGHTED FUNCTION SPACES: COMPACT EMBEDDINGS, TRACES AND DICHOTOMY, LOCAL SINGULARITIES, AND APPLICATIONS

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We study weighted spaces of Besov and Triebel-Lizorkin type where the weight belongs to some Muckenhoupt class. We investigate compact embeddings between such spaces characterised in terms of their entropy and approximation numbers, and present applications of our results to spectral theory. Another topic of our interest concerns traces on special (fractal) sets, and related dichotomy questions (i.e., whether the corresponding spaces admit traces, or alternatively the test functions supported outside this set are dense in the space). Finally we characterise the local singularity behaviour in such weighted spaces in terms of their growth and continuity envelopes.

Partly, this is based on joint work with L. Skrzypczak (Poznan) and H.-J. Schmeisser (Jena).

LOWER BOUNDS FOR SUMS OF EIGENVALUES OF ELLIPTIC OPERATORS AND SYSTEMS

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We prove Berezin-Li-Yau-type lower bounds with additional term for the eigenvalues of the Stokes operator and the Dirichlet Laplacian. Generalizations to higher-order operators and applications to the Navier-Stokes equations are also given.

STURM-LIOUVILLE OPERATORS WITH SINGULARITIES AND INDEFINITE INNER PRODUCTS

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We consider some Sturm-Liouville operators with singular potentials which generate self-adjoint operators in a Pontryagin space.

GENERALIZED TRIGONOMETRIC FUNCTIONS FROM DIFFERENT POINTS OF VIEW

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Generalized trigonometric functions have quite long history. The first version of generalized trigonometric was introduced in 1879 in work of Lundberg and since then generalized trigonometric functions were introduced/re-introduce in different areas of mathematics.

In this review we shall focus on impact of generalized trigonometric functions on different areas of analysis and on the roles they play in various branches of mathematics. We will explore generalized trigonometric functions from different standpoints. We will start from the analytic point of view and for each $p \in (1, \infty)$ introduce a function $\sin_p^{-1}$ by an integral formula, which is just an extension of the well known integral representation of arcsin, and then use it to define generalized sine, cosine and tangent functions (labeled $\sin_p$, $\cos_p$ and $\tan_p$ respectively). Numerous properties of these functions, such as an identity of Pythagorean type, are exhibited. Then we consider the unit circle in $\mathbf{R}^2$ with the l_p norm and define generalized trigonometric functions as is done in the standard case when the l_2 norm is used. We show that these functions coincide with those introduced earlier. In the third section we consider the integral operator $T : L^p(I) \to L^p(I)$ given by $Tf(x) = \int_0^x f(t)dt$, where $I = (0,1)$, and look at the problem of finding an extremal function (an element of the unit sphere of $L^p(I)$ at which the norm of T is attained). It turns out that the extremal functions are given by $\cos_p$. The following section deals with the Dirichlet eigenvalue problem for the $p-$Laplacian on a bounded interval: all eigenfunctions are expressible by means of $\sin_p$ functions, which corresponds exactly to the classical situation when $p = 2$. After establishing a connection with approximation theory and s-numbers, we conclude with a review of other ways in which the classical trigonometric functions have been generalized.

THE HARDY INEQUALITY AND CURVATURE

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Recent results will be presented for the Hardy inequality that include curvature considerations. Convex and non-convex domains will be considered.

SOBOLEV-TYPE EMBEDDINGS INTO GENERALIZED HÖLDER SPACES

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We present a sharp estimate of k-modulus of smoothness of a function f such that the norm of its distributional gradient $|\nabla^k f|$ belongs locally to the Lorentz space $L^{n/k,1}(\mathbb{R}^n)$, $k \in \mathbb{N}$, $k < n$, and its reverse form. Results are applied to establish necessary and sufficient conditions for continuous embeddings of Sobolev-type spaces, modelled upon rearrangement invariant Banach function spaces $X(\mathbb{R}^n)$, into generalized Hölder spaces defined by means of the k-modulus of smoothness ($k \in \mathbb{N}$).

The lecture is based on a joint research with Amiran Gogatishvili, Susana Moura and Júlio S. Neves.

Optimality and iteration

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We study a question whether sharp Sobolev embeddings can be iterated without losing sharpness. Examples show that sharpness can be lost, especially in limiting cases, when one is restricted for example to Lebesgue spaces or to Orlicz spaces. We focus on question whether optimality can be preserved when at each step one considers the optimal rearrangement-invariant Banach function partner spaces. We develop a general method which shows that the answer is positive under certain restrictions. We apply our results to Euclidean–Sobolev embeddings and to the Gaussian–Sobolev embeddings. In the case of the Gaussian–Sobolev embeddings, we reduce higher-order embeddings to boundedness of kernel operators and obtain thereby results that had not been known before. In the case of the Euclidean–Sobolev embeddings, we recover results that had been known by a new and rather elementary proof. This is a joint work with Andrea Cianchi.

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Integral operators on the semiaxis: boundedness and compactness, estimates of the approximation numbers

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We give a short survey on the boundedness and compactness as well as the sharp two-sided estimates of the singular, approximation and entropy numbers of some classes of integral operators on the semiaxis. It concerns asymptotic behavior and the Schatten-von Neumann norm.

SPECTRAL THEORY OF MATRIX DIFFERENTIAL OPERATORS

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In this talk methods for studying spectral properties of matrix differential operators will be presented. Special emphasis is placed on classes of operators to which standard results do not readily apply. The questions to be addressed include, in particular, the localization and structure of the spectrum. Applications to operator matrices arising in quantum mechanics such as Dirac operators, multi-channel Hamiltonians and the Klein-Gordon equation will be given.

SPECTRAL THEORY VIA OPERATOR M-FUNCTIONS - FORWARD AND INVERSE PROBLEMS

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A useful tool in studying forward and inverse problems for ODEs is given by the Weyl-Titchmarsh m-function. In PDE problems, a similar role is played by the Dirichlet-to-Neumann map. Both of these can be determined solely from the boundary behaviour of solutions. In this talk, we will look at some results in this area, before focussing on extending m-functions and Dirichlet-to-Neumann maps to the abstract setting of boundary triples, giving rise to operator M-functions. We will discuss properties of M-functions and their relation to the resolvent and the spectrum of the associated operator. In the symmetric case, it is known that the M-function contains all spectral information if the underlying minimal operator is simple. We will look at attempts to extend this kind of result to non-symmetric cases by considering some examples.