Inverse Problems Network Meeting 1

Monday, 24th April 2017 - Tuesday, 25th April 2017

WX/3.14, Queens Buildings, Cardiff University

Abstract of Talk

ON FOUNDATIONAL COMPUTATIONAL BARRIERS IN l^1 AND TOTAL VARIATION REGULARISATION IN INVERSE PROBLEMS

Anders Hansen
Cambridge University

The use of regularisation techniques such as l^1 and Total Variation in Basis Pursuit and Lasso for inverse problems has been a great success over the last decades. In this talk we will discuss universal boundaries regarding the existence of algorithms for solving these problems. For example we have the following paradox: it is impossible to design algorithms to solve these general problems accurately when given inaccurate input data, even when the inaccuracies can be made arbitrarily small. As a simple number such as $\sqrt{2}$ never comes with an exact numerical representation, inaccurate data input is a daily encounter. The impossibility result implies that for any algorithm designed to solve these problems there will be cases where the algorithm fails in the following way: For fixed dimensions and any small accuracy parameter $\epsilon > 0$, one can choose an arbitrary large time T and find an input such that the algorithm will run for longer than T and still not have reached ϵ accuracy. Moreover, it is impossible to determine when the algorithm should halt to achieve an ϵ accurate solution, and hence the algorithm will never be able to produce an output where one knows that the output is at least ϵ accurate. The largest ϵ for which this failure happens is called the Breakdown-ϵ. For Basis Pursuit and Lasso, the Breakdown-$\epsilon > \frac{1}{3}$ even when the absolute value of the input is bounded by one and is well conditioned.

The paradox opens up for a new classification theory to determine the boundaries of what computers can achieve in regularisation and inverse problems, and to explain why empirically many modern algorithms for solving regularisation problem in real-world scenarios perform very well. We will discuss positive classification results showing that sparse problems can be computed accurately. However, this is delicate; e.g. given standard assumptions from sparse recovery, there are algorithms that can compute a solution to Basis Pursuit accurately, however, this is impossible for Lasso and Basis Pursuit with noise parameter $\delta > 0$. However, one can compute a solution accurately up to the Breakdown-ϵ that tends to zero when δ tends to zero, and coincides with the error bound provided in the theory of sparse recovery. This helps explaining the success of many modern algorithms applied in numerous real-world scenarios, and also explains the cases where algorithms will fail and why.