VARATIONAL GAUSSIAN APPROXIMATION FOR POISSON DATA

Bangti Jin
University College London

The Poisson model is frequently employed to describe count data, but in a Bayesian context it leads to a numerically intractable posterior probability distribution. In this talk, I will discuss a variational Gaussian approximation for approximating the posterior distribution arising from the Poisson model with a Gaussian prior. This is achieved by seeking an optimal Gaussian distribution minimizing the Kullback-Leibler divergence, or equivalently maximizing the lower bound for the model evidence. We shall derive an explicit expression for the lower bound, and establish the existence and uniqueness of the optimal Gaussian approximation. The lower bound functional can be viewed as a variant of classical Tikhonov regularization, but penalizing also the covariance of the estimate, instead of only the mean. We develop an efficient alternating direction maximization algorithm for solving the problem, and discuss the strategies for reducing the computational complexity via low rank structure of the forward operator and the sparsity of the covariance. Further, as an illustration of the use of the lower bound, we discuss the hierarchical Bayesian modeling for automatically selecting the hyperparameter in the prior distribution, and propose a monotonically convergent algorithm for determining the hyperparameter. We present extensive numerical experiments to illustrate the Gaussian approximation and the algorithms.