Regularization is a classical technique for solving ill-posed inverse problems, but it is usually imposed in the discrete domain. In this talk, we consider a continuous-world scenario where an unknown function $f$ is probed with a finite number of linear functionals (forward imaging model) corrupted by measurement noise. The non-conventional aspect is our use of a continuous-domain regularization involving the $L_p$ norm of $L f$ where $L$ is a suitable differential operator. We present two representer theorems that provide the parametric form of the solution(s) of the reconstruction problem with Tikhonov ($p = 2$) vs. total-variation ($p = 1$) regularization. Remarkably, the solutions of both problems are (generalized) splines with the knots being fixed for $p = 2$ and adaptive (and fewer) in the total variation scenario. These findings suggest an exact discretization of the problem that can then be solved using finite-dimensional minimization techniques. We illustrate the theory with examples of signal reconstruction that confirm the sparsifying (resp., smoothing) effect of total variation vs. Tikhonov regularization.

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