Inverse Problems Network Meeting 3

Thursday, 26th April 2018 - Friday, 27th April 2018

Centre for Inverse Problems, UCL

Abstracts of Talks

FINITE ELEMENT METHODS WITH WEAKLY CONSISTENT REGULARISATION FOR INVERSE PROBLEMS

Erik Burman
UCL

The classical way of approximating inverse problems is to first regularise the continuous problem, ensuring well-posedness and then approximate the resulting well-posed problem using standard techniques. Although conceptually straightforward and relatively easy to implement, this approach leads to a nontrivial matching problem for the discretisation and regularisation parameters. Typically the regularisation parameter is chosen matching perturbations in data. The mesh size is then chosen small enough so that all scales of the regularised problem is resolved. In this talk we will discuss an alternative approach where the ill-posed problem (i.e. without regularisation) is discretised using finite elements in an optimisation framework. To counter the severe ill-posedness of the resulting finite dimensional system we add weakly consistent regularisation terms drawing on known results from computational methods for fluids. We show how the resulting methods can be analysed, yielding optimal error estimates, using the numerical stability of the regularised finite element methods in combination with sharp conditional stability estimates derived from Carleman estimates. Both the effect of discretisation and perturbations in data are included in the estimates. Poisson's equation will be used as model problem in the stationary case and in the transient case we will consider the heat equation, some numerical examples will be given illustrating the theory.

NONLINEAR RESPONSES FROM THE INTERACTION OF TWO PROGRESSING WAVES AT AN INTERFACE AND ASSOCIATED INVERSE PROBLEM

Maarten de Hoop
Rice University

For scalar semilinear wave equations, we analyze the interaction of two (distorted) plane waves at an interface between media of different nonlinear properties. We establish that new waves are generated from the nonlinear interactions. Furthermore, we show that the incident waves and the nonlinear responses determine the location of the interface and some information of the nonlinear properties of the media. In particular, for the case of a jump discontinuity at the interface, we can determine the magnitude of the jump in the nonlinear parameter. We will briefly indicate the generalization to nonlinear elastodynamics with the nonlinear interaction of two progressing polarized waves.

Joint research with Gunther Uhlmann and Yiran Wang.

INVERSE PROBLEM FOR THE HELMHOLTZ EQUATION WITH CAUCHY DATA: STABILITY AND RECONSTRUCTION

Romina Gaburro
University of Limerick

We study the performance of Full Waveform Inversion (FWI) from time-harmonic Cauchy data via conditional well-posedness driven iterative regularization. The Cauchy data can be obtained with dual sensors measuring the pressure and the normal velocity. We define a novel misfit functional which, adapted to the Cauchy data, allows the independent location of experimental and computational sources. The conditional well-posedness is obtained for a hierarchy of subspaces in which the inverse problem with partial data is Lipschitz stable. Here, these subspaces yield piecewise linear representations of the wave speed on given domain partitions. This is joint work with Giovanni Alessandrini, Maarten de Hoop, Florian Faucher and Eva Sincich.

DOMAIN-DECOMPOSITION PRECONDITIONING FOR LINEAR FREQUENCY DOMAIN WAVE PROBLEMS

Ivan Graham
University of Bath

There is currently large research interest in finding optimal solvers for finite-element discretisations of frequency-domain wave problems, such as the Helmholtz and time-harmonic Maxwell equations, when the frequency is large. Ideally such solvers should also have good parallel scaling properties, be robust to heterogeneities in material coefficients, and come with theorems rigorously justifying their behaviour.

Such problems occur regularly as the forward problem in practical inverse problems. For example the heterogeneous Helmholtz equation (and the corresponding frequency domain elastic wave equation) arise in seismic imaging (“full waveform inversion”).

A common approach to this problem is trying to find good preconditioners to use when solving the linear systems with (F)GMRES.

This talk will be about preconditioners built using (i) variants of classical additive-Schwarz domain-decomposition methods, and (ii) artificial absorption (similar to the ”shifted Laplacian” preconditioner involving multigrid).

The overall philosophy is to use, as much as possilble, PDE theory of the underlying boundary-value problems to tackle this linear-algebra problem of developing fast solvers and to use the flexibility of domain decomposition methods in order to choose subdomain solvers which are suitable for solving wave propagation problems.

The work on Helmholtz is joint with Eero Vainikko (Tartu), and Jun Zou (Chinese University of Hong Kong).

The work on Maxwell is joint with Marcella Bonazzoli (Paris 6), Victorita Dolean (Strathclyde/Cote dÁzur), and Pierre-Henri Tournier (Paris 6).

GENERALIZED LINEARIZATION IN ELECTRICAL IMPEDANCE TOMOGRAPHY

Nuutti Hyvönen
Aalto University

Electrical impedance tomography aims at reconstructing the interior electrical conductivity from surface measurements of currents and voltages. As the current-voltage pairs depend nonlinearly on the conductivity, impedance tomography leads to a nonlinear inverse problem. Often, the forward problem is linearized with respect to the conductivity and the resulting linear inverse problem is regarded as a subproblem in an iterative algorithm or as a simple reconstruction method as such. In this paper, we compare this basic linearization approach to linearizations with respect to the resistivity or the logarithm of the conductivity. It is numerically demonstrated that the conductivity linearization often results in compromised accuracy. Inspired by these observations, we present and analyze a new linearization technique which is based on the logarithm of the Neumann-to-Dirichlet operator. The method is directly applicable to discrete settings, including the complete electrode model. We also consider Frechet derivatives of the logarithmic operators. Numerical examples indicate that the proposed method is an accurate way of linearizing the problem of electrical impedance tomography.

TOWARDS IMPROVED METAL DETECTION USING (GENERALISED) MAGNETIC POLARIZABILITY TENSORS

Paul Ledger
Swansea University

Locating and identifying hidden conducting objects has a range of important applications including searching for buried treasure, identifying landmines and in the early detection of concealed terrorist threats. Traditional approaches to the metal detection problem involve determining the conductivity and permeability distributions in the eddy current approximation of Maxwell's equations and lead to an ill-posed inverse problem. On the other hand, practical engineering solutions in hand held metal detectors use simple thresholding and are not able to discriminate between small objects close to the surface and larger objects buried deeper underground.

In this talk, an alternative approach in which prior information about the form of the conducting object has been introduced will be discussed. This allows the perturbed magnetic field, due to the presence of a conducting (permeable) object, to be described in the form of an asymptotic expansion as the object size tends to zero. The asymptotic expansion separates the object's position from its shape and material description offering considerable advantages in case of isolated objects. Our previous result focused on the leading order term and described the object using a rank 2 magnetic polarizability tensor (MPT). The coefficients of the MPT can be computed by solving a vector valued transmission problem numerically using finite elements. Recently, we have extended this result by providing a new complete asymptotic expansion of the perturbed magnetic field as the object size goes to zero. In this expansion, the object is described in terms of a new class of generalised MPTs (GMPTs), which can also be computed by solving vector valued transmission problems. We believe that our new result will have important implications for metal detectors since it will improve small object discrimination and, for situations where the background field varies over the object, this information will useable, and indeed useful, in characterising objects.

The talk will explore the interesting properties exhibited by (G)MPT, which characterise conducting objects. It will describe how the eigenvalues of the MPTs for candidate target objects can form a dictionary for object classification in an off-line stage. Initial investigations using a machine-learning approach to object classification, using the aforementioned dictionary, will also be included.

MACHINE LEARNING FOR INVERSE PROBLEMS

Peter Maass
University of Bremen

The classical approach to inverse problems starts with an analytical description $F : X \to Y$ of the forward operator in some function spaces X, Y. The field of inverse problems addresses the task of approximating an unknown x^* from noisy data $y^\delta \sim F(x^*)$ with the further complication that F^{-1} or any type of generalized inverse is unbounded. The mathematical analysis stays within this framework and provides a regularization theory for optimal analytical convergence rates, stability estimates and convergence of numerical schemes.

This model driven approach has at least two shortcomings. First of all, the mathematical model is never complete. Extending the model might be challenging due to an only partial understanding of the underlying physical or technical setting. Secondly, most applications will have inputs which do not cover the full space X but stem from an unknown subset or obey an unknown stochastic distribution. E.g. there is no satisfactory mathematical model, which characterizes tomographic images or other image classes amongst all L_2-functions.

Machine learning offers several approaches for amending such analytical models by a data driven approach. Based on sets of training data either a specific problem adapted operator update is constructed and an established inversion process is used for regularizing the updated operator or the inverse problems is addressed by a machine learning method directly. We present an overview on machine learning approaches for inverse problems. We include some first numerical experiments on how to apply deep learning concepts to inverse problems and we finsih by showing some first applications of data driven model updates for magentic particle imaging (MPI).

AN OVERVIEW OF THE LINEAR SAMPLING METHOD IN THE TIME DOMAIN AND ITS ANALYSIS FOR IMAGING PENETRABLE OBSTACLES

Virginia Selgas
Universidad de Oviedo

This is joint work with F. Cakoni (University of Rutgers) and P. Monk (University of Delaware).

We consider the problem of locating and reconstructing the geometry of an obstacle from time domain measurements of causal waves. More precisely, we assume that the incident fields are due to causal point sources placed on a surface, and that the corresponding scattered fields are measured on the same surface. To determine the position and shape of the target from such multi-static scattering data, we propose to apply the Time Domain Linear Sampling Method (TDLSM).

We start by motivating this inverse problem in different scenarios, such as acoustic waveguides, and showing the performance of TDLSM there. We then focus on the case of a penetrable obstacle and explain the difficulties arising in the analysis of the TDLSM. In this respect, we propose a new study of the TDLSM based on localizing the interior transmission eigenvalues in the Fourier-Laplace domain; by doing so, we prove that the TDLSM for penetrable targets has similar blow-up properties to its usual frequency domain version. We also prove new time domain estimates for the forward problem and the interior transmission problem, as well as analyze several time domain operators arising in the inversion scheme.