Inverse Problems Network Meeting 4

Thursday, 17th January 2019 - Friday, 18th January 2019

Mall Room, Level 8 of the School of Mathematics, University of Leeds

Abstracts of Talks

IMAGING SMALL POLARIZABLE SCATTERERS WITH POLARIZATION DATA

Dr M Cassier
Fresnel Institute

Joint work with P. Bardsley (Cirrus Logic) and F. Guevara Vasquez (University of Utah)

In this talk, we present a method for imaging small scatterers in a homogeneous medium from polarization measurements of the electric field at an array. The electric field comes from illuminating the scatterers with a point source with known location and polarization. We view this problem as a generalized phase retrieval problem with data being the coherency matrix or Stokes parameters of the electric field at the array. We introduce a simple preprocessing of the coherency matrix data that partially recovers the ideal data where all the components of the electric field are known for different source dipole moments. We show that the images obtained using an electromagnetic version of Kirchhoff migration applied to the partial data are, for high frequencies, asymptotically identical to the images obtained from ideal data. We analyze the image resolution in the Fraunhofer regime and show that polarizability tensor components in an appropriate basis can be recovered from the Kirchhoff images, which are tensor fields. A time domain interpretation of this imaging problem is provided and numerical experiments are used to illustrate the theory.

The work of Maxence Cassier was supported in part by Simons Foundation grant #376319 (Michael I. Weinstein).

DATA RECONSTRUCTION FOR THE BIHARMONIC PLATE EQUATION

Dr. T Johansson
Aston University

We consider an incomplete boundary data problem for the biharmonic plate equation, where the displacement is known throughout the boundary of the plate but the normal derivative and bending moment are given on only a portion of the boundary. This renders an inverse ill-posed problem. An iterative regularizing method will be presented together with some of its properties, notably convergence. Numerical realisation of the given scheme is included as well.

THE ALTERNATING METHOD FOR CAUCHY PROBLEMS

Prof. V Kozlov
Linköping University

The alternating method for solving elliptic problems with Cauchy data given on a part of the boundary consists of iterative solutions of Dirichlet - Neumann mixed boundary value problems for the same elliptic equation. Its convergence was proved in 1990 by V. Maz'ya and the author. I will talk about further development of this method and its applications. In particular about its modification (Dirichlet - Robin iterations) for solving the Helmholtz equation, its connection with the Landweber iterations, various accelerations and other extensions.

HISTOGRAM TOMOGRAPHY

Prof. W Lionheart
University of Manchester

In many tomographic imaging problems inverse problems the data are integrals over lines of scalar, vector or tensor fields. In this talk we consider a range of problems in which for each line one has information about the distribution of the values along the line. In the scalar case one application is near infrared spectral tomography, for vector fields Doppler velocimetry and for tensor fields examples include strain tomography from neutrons or x-rays. We will look at two separate approaches to reconstruction, one using the cumulative distribution to reconstruct the characteristic function of sub-level sets and another using moments of the distribution.

INVERSE PROBLEMS IN MANUFACTURING, ROBOTICS, AND THE NUCLEAR INDUSTRY

Prof A Mulholland
University of Strathclyde

I will discuss an inverse problem whose input arises from an array of sensors, whose goal is to reconstruct a spatial map of some material property within a domain, and whose purpose is to reduce the uncertainty in some measurement. I will discuss two application areas that at first sight might appear to be unconnected but the underlying problem and the method of solution are in fact common. The first application area is in the nondestructive imaging and measurement of flaws in safety critical structures such as those found in the nuclear industry. Traditional imaging algorithms within ultrasonic non-destructive testing (the dominant sensing modality in this area by far) typically assume that the material being inspected is homogeneous. Often the medium is in fact spatially heterogeneous and this leads to poor detection, sizing and characterisation of defects. I will discuss our recent work which reconstructs the inhomogeneous wave speed maps of random media from ultrasonic phased array data. This is achieved via application of the reversible-jump Markov chain Monte Carlo method to a Fullwave Inversion methodology. The inverted maps are used in conjunction with an imaging algorithm to correct for deviations in the wave speed, and the reconstructed flaw images are then used to quantitatively assess the success of this methodology. I will also show how this approach can be used to optimise the design of a new component at the manufacturing stage such that it maximises the ability thereafter to test this component non-destructively for the presence of any flaws. In a second application area I will describe how modern manufacturing increasingly utilizes automated systems for component positioning and assembly. A vital aspect of autonomous precision manufacturing is large volume metrology (so a local GPS system for robots if you prefer). One popular approach uses light rays, which travel through the volume of air, to calculate the position of an object of interest. These optical-based metrology systems such as photogrammetry and laser tracking are crucial in improving the accuracy and quality associated with robotic assembly. In an industrial setting these positional measurement systems give rise to uncertainties which can in many instances be greater than the required tolerances. One source of this uncertainty is light refraction due (in part) to temperature fluctuations in the air. The inverse problem associated with using light-based sensor data to reconstruct the refractive index map in the spatial domain volume can then be tackled using a similar methodology as above. I will show how this reduces the measurement uncertainty and facilitates the use of these light based systems in assisting robotic manufacturing.

CONTROLLING SOLUTIONS OF ELLIPTIC PDES, AND APPLICATIONS TO INVERSE PROBLEMS

Prof M Salo
University of Jyväskylä

The classical Runge approximation theorem states that an analytic function in a simply connected domain in the plane can be approximated locally uniformly by global analytic functions. More generally, it was observed by Lax and Malgrange in the 1950s that solutions of an elliptic PDE in a subdomain can be approximated by solutions in a larger domain, provided that a certain form of the unique continuation principle holds. These approximation properties can be used to control the profile and energy of solutions on certain parts of the domain, and they have been used in several ways in inverse problems. In this talk we will discuss some recent uses of the approximation property in inverse problems, including monotonicity methods for shape detection, stability analysis, and inverse problems for fractional equations.

INVERSE OBSTACLE SCATTERING PROBLEMS: A GAME OF HIDE AND SEEK

Prof. B Sleeman
University of Dundee and University of Leeds

We review the problem of establishing uniqueness of construction of scattering obstacles under a variety of far field data. A discussion of approximate cloaking is presented and we consider whether it is possible to detect the presence of a cloak.

SPECTRUM AND A TRACE FORMULA FOR STATIONARY SPACETIMES

Prof. A Strohmaier
University of Leeds

We consider a natural spectral problem appearing in the analysis of wave equations on stationary spacetimes. It turns out that the spectrum in such a relativistic context is not the spectrum of a self-adjoint operator, but rather the spectrum of an operator pencil, or, somewhat equivalently, the spectrum of a Krein-self-adjoint operator on a Pontryagin space. We prove a trace formula and a singularity expansion for the spectrum. The underlying geometry is the symplectic geometry of the space of lightlike geodesics. This has applications to the inverse problems of detecting properties of the spacetime from the spectrum. (joint work with S. Zelditch)